UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Matematica I

1920-1-E2702Q001

Aims

To understand the ideas and techniques of differential and integral calculus for real functions of one real variable.

Contents

Real numbers, operations and their properties. Elementary functions, properties and their graphs. Numerical sequences, limits of sequences, forms of indecision. Comparison of infinities. Numerical series, convergence tests. Absolute convergence. Function limits. Continuity. The derivative. Theorems of differential calculus. The Taylor's theorem. Primitive functions and indefinite integral.The Riemann Integral. Generalized integrals.

Detailed program

Real numbers. Natural numbers N, relative integers Z. The field Q of rational numbers. Properties and their inadequatecy: the equation x ^ 2 = 2 has no solution in Q * (demonstration "for absurd"). The real numbers field R. Decimal representation. The real axes, ordering. Intervals. Neighbourhood. Absolute value. Bounded sets in R. Maximum and minimum. Supremum and infimum, completeness. Roots, powers and logarithms. Induction principle.

Real functions of a real variable. Definition. Domain and range. Graph of a function. Elementary functions: powers, exponentials, logarithmic functions. The sequence as a function whose domain is the set N. Bounded function. Maximum, minimum, superior, inferior of a function. Properties of a real function: injectivity, increasing, decreasing, monotone, convex, concave, even, odd. Extremal points, absolute minimum or relative (extreme). Recognition of the given definitions by reading the graph. Composite function, inverse function. Periodic functions, trigonometric functions and their inverse. Solving the inequalities by inversion of injective and monotonic functions.

Complex numbers. The C field of complex numbers: algebraic form, operations, equality. Representation in the complex plane. Polar coordinates, modulo and argument, trigonometric form, equality. De Moivre's formula. Formula of the n-th roots of a complex number * (solutions in C of the equation z ^ n = w). The fundamental theorem of algebra.

Limits. Limits of sequence, of functions. Properties: uniqueness of the limit *, permanence of the sign *, existence of the limit for monotonic functions (proved for sequences *). Comparison test. The ratio test. Operations with limits, indecision forms. The limit e. Notable limits. Symbol of asymptotic Landau. Order of an infinitesimal / infinite, compared to a sample.

Numerical series. Sequence of partial sums. Convergent, divergent, irregular series. Geometric * series, Mengoli series, harmonic series. Necessary condition of convergence *. Series with positive terms: their regularity * and convergence tests: comparison *, asymptotic comparison, root * and ratio tests. Series with alternating signs and Leibniz test. Simple and absolute convergence.

Continuity. Continuous function at a point, on a set. Classification of discontinuities. Operations between continuous functions, continuity of composite function. Properties of continuous functions in a closed and bounded interval: Weierstrass theorem, existence of zeros *, Darboux (or intermediate values) *. Continuity and monotony. Continuity of the inverse function.

Differential calculus. The derivative and its geometric interpretation. Equivalence between derivability and differentiability for functions of a variable. Equation of the tangent line. Points of non-derivability. Continuity and derivability *. Calculation rules for derivatives. Stationary points. Theorems of the differential calculus: Fermat *, Rolle *, Lagrange * and its corollaries *, examples and counterexamples. The theorems of De l'Hòpital. Derivatives of higher order. Polynomial approximation: Taylor formula, Peano remainder and Lagrange remainder. McLaurin's formulas. Convexity and inflection points. Use of the derivatives of higher order to establish the nature of a stationary point *. Asymptotes. Study of the graph of a function. Primitive functions and indefinite integral. Elementary methods for the search for a primitive. Integration by parts, by substitution (change of variable). Integration of rational functions.

The Riemann integral . Definition and its properties. The integral average theorem *. Integral function, integration and differentiation*. The fundamental theorem of calculus *. Generalized integrals, definitions and examples.

Prerequisites

Sets operations, union, intersection; membership and inclusion. Operations and comparison between real numbers, sorting. Properties of powers. Second-degree equations. Binomial expansion. Polynomials, division between polynomials, root of a polynomial, Ruffini's rule. Factoring. First and second degree inequations, rational inequalities. Cartesian coordinates. The line, the parabola, the circle. Degrees and radians. Elements of trigonometry. Systems of first degree equations.

Teaching form

In the classroom, teaching on the blackboard

Textbook and teaching resource

The students can choose one of the two proposed books:

--S.Salsa, C.Pagani, *Matematica. Per i diplomi universitari.* Zanichelli.

or,

--M. Conti, D.L. Ferrario, S. Terracini, G. Verzini: Analisi matematica, Vol I, dal calcolo all'analisi, Apogeo, 2006.

Notes, exercises sets and other material will be distributed. It will be available from the e-learning site.

Other exercises can be found on the book

--S.Salsa, A.Squellati, *Esercizi di Analisi matematica 1*, Zanichelli.

Semester

First Semester

Assessment method

Written examination with **optional oral colloquium.**

The goal of the evaluation (partial, complete and oral colloquium) is to ascertain a correct assimilation of concepts and techniques studied during lessons and exercises sessions.

The written exam is passed ONLY if the vote is greater or equal to 18/30.

The written exam will consist of exercises (similar to those done in the classroom and/or proposed to the students in the lectures) up to 24 points. There will be a maximum of 6 points for questions relating the theory (basic definitions and theoretical results done in the lectures).

Oral exam (optional)

Oral exam is not compulsory and will be done typically after a couple of days of the written exam. It is only possible to take the oral exam if the mark in the written part is greater than 18/30.

Students who got a positive grade in the written part (i.e., at least 18/30) might choose to take an oral exam to try to get a better grade if *they think that their preparation is good enough.* Needless to say, the oral exam can change the written grade in the positive, as well as in the negative direction. In particular, the minimal grade in the written part plus a poor oral part might end up in a failed exam.

The students who have not passed the written part, might be **alla prova orale e potranno sostenere essa SOLTANTO SE:**

-hanno avuto nella prova scritta un voto uguale o maggiore a **17/30, oppure**

- hanno avuto nella prova scritta un voto **uguale o maggiore a 16/30 di cui al meno 4/6 nelle domande teoriche.**

Partial exams (optional)

During the course there will be two partial exams. The that could allow the students to Students are not committed to do them, but in case they do they are allowed to skip the final complete written examination.

Office hours

By appoinment (via email)