UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Calcolo delle Probabilità

1920-3-E3501Q014

Aims

The course aims at providing students with the basic concepts and tools of probability theory, together with an illustration of some applications. At the end of the course students will have acquired the following:

- *knowledge*: language, definitions and statements of the fundamental results in probability theory;
- *competence*: operational understanding of the main proof techniques;
- *skills*: ability to apply theoretical notions to the solution of exercises and the analysis of problems.

Contents

1. Probability spaces
2. Random variables
3. Convergence of random variables
4. Introduction to Markov chains
5. Examples of probability models

Detailed program

1. *Probability spaces*

- Introduction to probability: mathematical models for a random experiment

- Axioms of probability
- Basic properties of probability, c
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- Conditional probability, Bayes theorem
- Independence of events, Bernoulli trials

2. *Random variables*

- Reminders of measure theory
- Important distributions, discrete and continuous, on the real line
- Random variables
- Marginal laws and joint law

- Independence of random variables
- Transformations of random variables
- Expected value, variance and covariance
- L^p spaces, inequalities (Jensen, Cauchy-Schwarz, Hölder)
- Correlation coefficient and linear regression (hints)

3. *Convergence of random variables*

- Reminder on convergence theorems in the theory of integration
- Borel-Cantelli lemma
- Weak and strong law of large numbers
- Notions of convergence for sequences of random variables (a.s., in probability, in L^p)
- Weak convergence of probabilities, convergence in law of random variables
- Law of small numers (weak convergence of the binomial to the Poisson distribution)
- Central limit theorem via Lindeberg's principle
- Central limit theorem via characteristic functions (hints)
- The method of normal approximation
- Independence of sigma-algebras, Kolmogorov's 0-1 law

4. *Introduction to Markov chains*

- Introduction to stochastic processes, finite-dimensional distributions
- Markov chains, transition matrix, Markov property
- Recurrent and transient states, invariant and reversible measures
- Convergence theorem (hints): convergence to equilibrium, law of large numbers
- Absorption probabilities (hints)
- Random walks on graphs (hints)

5. *Examples of probability models (presented alongside the theory)*

- Classical paradoxes (birthdays, Monty-Hall, Borel, Bertrand)

- Random permutation and fixed points
- Concentration properties of volume in high-dimensions
- Weierstrass' approximation theorem and the law of large numbers
- Simulation of random variables, the Monte Carlo method
- Simple random walk in one and more dimensions
- Gambler's ruin

- The PageRank algorithm

Prerequisites

The knowledge, competences and skills taught in the courses of the first two years, in particular *Linear Algebra, Analysis 1 and 2 (= calculus in one and more variables), Measure Theory*.

Teaching form

Lectures and recitations in the classroom, divided into:

- theoretical lectures (10 ects) focused on the knoledge of definitions, results and relevant examples, as well as the competences linked to their comprehension;
- recitations (2 ects) focused on the skills necessary to apply the theoretical knoledge and competencies to the solution of exercises.

The course is given in Italian.

Textbook and teaching resource

Reference textbooks

- F. Caravenna, P. Dai Pra. *Probabilità. Un'introduzione attraverso modelli e applicazioni.* Springer-Verlag Italia, Milano (2013).
- D. Williams. *Probability with Martingales.* Cambridge University Press (1991).

Other dydactical material (available on the e-learning page of the course)

- Notes by the teacher on specific arguments
- Weekly exercise sheets (with detailed solutions)
- Written exams from previous years (with detailed solutions)
- List of proofs for the oral examination

Semester

Third year, First (Fall) Semester.

Assessment method

Written examination (or midterms) and oral examination, with the rules described in the sequel. The aspects that will be evaluated are the correctness of the answers, the creativity, the precision, the clarity of exposition. There will be 5 exam sessions (two in February, one in June, one in July, one in September).

- The *written examination* lasts 3 hours and gets a mark out of 30. This examination tests practical skills (solving exercises) and also theotetical knoledge and competencies (definitions, examples and counter-examples). The written examination is passed with a minimal mark of 15/30 and allows to be admitted to the oral examination.
- In the middle and at the end of the course there will be two *midterm written exams,* which last 1.5 hours each and get a mark out of 15. Passing both midterms with a minimal mark of 7,5/15 is equivalent to passing the written examination (with the "sum" of the marks) and allows to be admitted to the oral examination.

- The *oral examination* lasts 30-45 minutes and gets a mark out of 30. It can be given (after passing the written examination) in any exam session of the same academic year. The oral examination tests the knowledge of a selection of proofs as well as a working knowledge of the notions of the course. The oral examinations is passed with a minimal mark of 15/30.
- The final mark results from the average between the marks of the written and oral examinations. The exam is passed with a minimal mark of 18/30.

Exemption from the oral examination. Passing the written examination with a mark in the range 20-27/30 allows to be exempted from the oral examination, the final mark being equal to the mark obtained in the written examination; with a mark greater than 27/30 it is still possible to be exempted from the oral examination, however the final mark in this case will be 27/30; finally, with a mark smaller than 20/30 it is necessary to take the oral examination.

Office hours

To be fixed at the beginning of the course and communicated in the e-learning page.