UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Geometry III

1920-3-E3501Q055

Aims

The aim of the course is to introduce the study of topological spaces by means of their most basic algebraic invariant, that is, the first fundamental group, and to develop the foundations of the theory of differential forms on differential manifolds, which is a much more general and flexible framework than the one considered in Geometry II.

The expected learning outcomes include the following:

- the knowledge and understanding of the basic definitions and statements, as well as of the basic strategies of proof in the theory of first fundamental group and of differential forms; the knowledge and understanding of some of the most relevant basic applications, notably to the study of smooth proper maps between differentiable manifolds; the knowledge and understanding of some of the key foundational examples of the theory;
- the ability to apply the acquired abstract knowledge to the solution of simple computational exercises and theoretical problems, referring in a precise and well-organized manner to the pertinent results being used; the ability to master the algebraic, differential and integral calculus of differential forms, and to use it in the some simple practical situations, such as the study of the first fundamental group of some simples spaces and of proper maps between differentiable manifolds; the ability to apply the theoretical background to the construction and discussion of simple examples and solution of exercises; the ability to expose and communicate effectively and clearly the theoretical content of the course.

Contents

Topological coverings and first fundamental group; differentiable manifolds; tangent bundles; vector fields and their associated flows; differentiable manifolds; Stokes ' Theorem; De Rham theory; proper maps and degree theory.

Detailed program

Topological coverings and the universal cover; first fundamental group; lifting theorems; Seifert-Van Kampen Theorem; differentiable manifolds; vector fields and tangent bundles; differential forms; pull-back and differential; exterior derivative; Lie derivative and Cartan's magic formula; oriented manifolds and integration; smooth domains and Stokes' Theorem; proper maps and degree theory; proper homotopies; applications (e.g., retractions, vector fields on spheres).

Prerequisites

The content of the courses of Analysis I and II, Linear Algebra and Geometry, Geometry I and II.

Teaching form

Lectures: 6 CFU

Textbook and teaching resource

Reference text: teacher's notes on e-learning

Recommended reading:

the following books are especially pertinent to the content of this course:

- V. Guillemin and P. Haine, Differential forms, World Scientific 2019
- W. Fulton, Differential Topology, a first course, Springer Verlag 1995

· M. Do Carmo, Differential forms and applications, Springer Verlag 1996;

· V. Guillemin, A. Pollack, Differential Topology 1974;

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Semester

Assessment method

During the course, two written partial tests will be offered, each referred to one half of the course. Each partial test will consist of a balanced flexible combination of computational exercises and theoretical questions. The exercises and theoretical questions in these tests will be along the lines of those offered in the practical and theoretical tests of the regular exam sessions (see below). The two partial tests will contribute equally to the final grade. To pass the exam through the partial tests, the student needs to pass each of them, thus obtaining a grade of at least 18/30 in both.

Alternatively, students may pass the exam through the regular exam sessions that follow the end of the course, and exactly the same pattern will be offered in every exam session. Thus, each session comprises two written tests, each referred to one half of the course, and consisting of a balanced combination of computational exercises and theoretical questions. The theoretical questions will involve definitions, statements of theorems, proofs, construction of examples and counterexamples, and simple theoretical problems.

The exercises will measure the student's ability to master the acquired formalism and apply it to some simple computations, to build on the acquired theoretical knowledge, and to invoke it in a pertinent and precise manner.

The theoretical questions will evaluate the knowledge and understanding of the conceptual framework of the course, as well as the ability to expose it in a well-organized, consistent and effective manner.

In order to pass the exam in one of the regular sessions, the student needs to obtain a grade of at least 18/30 in each of the two texts, which will contribute equally to the final grade. The two tests needn't be undertaken in the same session. It is also allowed to pass one the tests during the course and the other in a regular exam session.

To each exercise/theoretical question (or problem) a maximum partial grade will be assigned by the commission, depending on its difficulty and length. In the evaluation, every student will be given a grade in correspondence to each exercise/theoretical question (or problem) up to the maximum one, measuring the exactness, the completeness, the rigour, the clarity and the overall coherence of the development.

The exact subdivision of the course in two parts will be communicated well in advance during its duration.

Office hours

Upon appointment.