UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Measure Theory

1920-2-E3501Q053

Aims

The students will learn the theoretical aspects and the basic analytic applications of Measure and Intrgration Theory.

They will have to understand and be able to present with clarity and rigour the main definitions, theorems and proofs. They also should be able to use what they have learned to solve exercises and problems given during class and at the exam.

Finally they should be ready to apply the ideas and techniques of Measure Theory to the various mathematical and scientific fields where they play a role.

Contents

- The Riemann integral and the problems with passing to the limit.
- Algebras, sigma-algebras and measures. Measurable functions.
- Outer measures, premeasures, extension theorem. Borel and Lebesgue measures.
- Abstract integration. Convergence theorems
- Integration in several variables. Fubini-Tonelli theorem. Change of variables.
- Completeness of L^1.

Detailed program

1. The Riemann integral (an overview) and the problems with passing to the limit. Need of an integral better suited to deal with pointwise convergence of sequences of functions. A possible approach and an obstacle: Vitali's set.

2. Abstract measure theory. Algebras, sigma-algebras and measures. Basic properties and examples. Complete measures. Borel sigma-algebra. Product of sigma-algebras. Measurable functions. Simple functions. Measurability of the pointwise limit of a sequence of measurable functions. Measurable functions as pointwise limit of simple functions.

3. How to construct relevant measures. Outer measures and a way to generate some of them. Caratheodory condition and theorem. Premeasures and the extension theorem. Borel and Lebesgue measures.

4. Abstract integration. Integration of non negative functions. Monotone convergence theorem, Fatou's Lemma. Integration of complex valued functions. The dominated convergence theorem.

5. Integration in several variables. Fubini-Tonelli theorem. Change of variables.

6. Completeness of L^1.

Prerequisites

The basic courses in Analysis of one and several real variables. A good knowledge of general topology and to be familiar with abstract algebra is also recommended.

Teaching form

Frontal teaching.

Textbook and teaching resource

Notes from the instructor, collections of previous written tests, teaching resource from past years.

Main reference text: Folland, Real Analysis, Wiley

Other texts:

- Ambrosio - Da Prato - Mennucci, Introduction to Measure Theory and Integration, Edizioni della Normale.

- Rudin, Real and Complex Analysis,
- Stein - Shakarchi, Real Analysis, Measure Theory, Integration and Hilbert spaces, Princeton

Semester

Spring semester. March-June 2019.

Assessment method

The exam consists of a written part and an oral part. The written part tests how well the student can use in problem solving what he/she has learned in the course. During the oral test, there will be a discussion of the written test and then the student will be asked to present with clarity and rigour some definitions and theorems with their proofs. Passing the written test is mandatory to be admitted to the oral test. The written test is past if the grade obtained is at least 18. In that case the student can choose in which session, within the same academic year, he/she prefers to take the oral test.

The student is admitted to the oral test also with grades 15, 16, 17, but in such a case, he/she MUST take it in the current session.

To pass the exam is required: knowledge and correct use of the convergence theorems; a clear and precise picture of the abstract theory of measure and integration; a good understanding of Borel and Lebesgue measures in one and higher dimensions. The grade will depend on how the student can state, use and prove the main theorems.

The written and oral tests have the same weight in determining the final grade.

There will be six exam sessions in one academic year: june, july, september, november, february/march, april.

Office hours

By appointment, mostly on thursdays 14.00 - 16.00.