UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Analisi Matematica I

1920-1-E3501Q001

Aims

- To understand the basic concepts and the rigorously developed theory of modern mathematical analysis for functions of a single real variable.
- To master the contents and the techniques in order to solve mathematical problems and to apply them to different contexts.
- To acquire the ability of independently make judgments in the application of the learned methodologies to the solution of mathematical problems.
- To be able to express in a precise, rigorous and exhaustive way both the acquired theoretical knowledge and the solutions, independently worked out, of exercises and problems.
- To be able to learn the contents of the following courses delivered within the Mathematics Degree Course.

Contents

Real and complex numbers. One-variable calculus: limits, continuity, differential calculus, integration. Sequences and series.

Detailed program

1. **Natural numbers.** Peano Axioms, Induction Principle, recursive definitions.

2. **Real numbers.** Field axioms, order axioms, rational numbers, the completeness axiom, Dedekind cuts. The Archimedean property of the real-number system. Supremum and infimum of a set, properties of the supremum and the infimum. Existence of roots of nonnegative real numbers. Rational and real powers.

Binary and decimal representation of real numbers.

3. **Complex numbers.** Definition, algebraic form, modulus, conjugate of a complex number, real part and imaginary part, triangle inequality. Trigonometric and exponential form of a complex number, products and power of complex numbers in trigonometric/exponential form. Complex exponentials. Roots of complex numbers. Fundamental theorem of algebra.

4. **Topology of the Real Numbers**. Distance, neighborhoods, interior points, boundary points. Open sets, closed sets. Accumulations points, isolated points. Density of rational numbers in the real numbers. Bolzano–Weierstrass Theorem.

5. **Functions.** Definition, domain, codomain, and range. Injective and surjective functions, bijections. Composition of functions, inverse functions, restriction. Countable sets. Countability of rational numbers and uncountability of irrational numbers. Real-valued functions of one real variable, the graph of a function. Monotonic functions, supremum and infimum, maximum and minimum. Elementary functions and their graphs (powers, exponentials, logarithms, trigonometric functions and their inverses, absolute value function, integer part, fractional part, sign function).

6. **Limits.** Definitions, examples, properties: uniqueness of the limit, Sign Permanence Theorem, Squeeze Theorem. Limit of sum, product, quotient and composition of functions. Special limits. One-side limits. Limits of monotonic functions. Landau symbols. Comparison of infinitesimals.

7. **Numerical sequences.** Limits of sequences. Boundedness of converging sequences. Subsequences. Existence of a convergent subsequence for a bounded sequence. Heine-Borel theorem. Monotonic sequences. The number *e.* Cauchy sequences. Upper and lower limits.

8. **Continuity.** The definition of continuity of a function. Composite functions and continuity. Sign Permanence Theorem. Bolzano's theorem. The intermediate-value theorem. Continuity of the inverse function. Continuity of elementary functions: powers, exponentials, logarithms, trigonometric functions and their inverses. Sequential criterion for the continuity of a function. Weierstrass theorem. Uniform continuity. Heine-Cantor theorem. Discontinuities. Lipschitz continuity.

9. **Series.** Definition. Convergent series, divergent series. Telescoping series, geometric series. Necessary condition for convergence of series. Absolute convergence. Series of nonnegative terms: comparison test, root test and ratio test. Alternating series: Leibniz's test.

10. **Differential calculus.** The derivative of a function. Geometric interpretation of the derivative as a slope. Left-hand and right-hand derivatives. Continuity of differentiable functions. The algebra of derivatives. The chain rule for differentiating composite functions. Derivatives of inverse functions. Derivatives of elementary functions. Extreme values of functions. Fermat's theorem. Rolle's theorem. The mean-value theorem for derivatives and applications. Relation between monotonicity and sign of the derivative. Cauchy's generalized mean value theorem. De l’Hôpital's rule. Convex and concave functions. The sign of the second derivative and the convexity/concavity of a function. Inflection points. Taylor's formula with Peano form of the remainder. Taylor's formula with mean-value form of the remainder.

11. **Integral calculus.** Step functions, definition of the integral for step functions. Properties of the integral of a step function. Upper and lower integrals on bounded intervals. Riemann integral. Properties of the Riemann integral (linearity, monotonicity). Integrability of the positive/negative part and of the modulus of an integrable function. Integrability of the restriction of an integrable function, integral over oriented intervals, additivity with respect to the interval of integration. Integrability of monotonic functions and continuous functions. Mean-value theorems for integrals. Fundamental theorem of calculus. Antiderivatives. Integration by parts, change of variable. Integration of rational functions. Improper integrals.

Prerequisites

Elementary algebra, elementary trigonometry, elementary analytic geometry.

Teaching form

Lessons (8 CFU), exercise classes (4 CFU).

The course is delivered in Italian

Textbook and teaching resource

Textbook: E. Giusti, Analisi Matematica I, Bollati Boringhieri.

Suggested readings:

- G. De Marco: Analisi Uno, Zanichelli Decibel.
- C. D. Pagani, S. Salsa: Analisi matematica 1, Zanichelli.

Exercise books:

- E. Giusti: Esercizi e complementi di analisi matematica, volume 1, Bollati Boringhieri.
- G. De Marco, C. Mariconda: Esercizi di calcolo in una variabile, Zanichelli Decibel.
- S. Salsa, A. Squellati: Esercizi di analisi matematica 1, Zanichelli.
- E. Acerbi, L. Modica, S. Spagnolo: Problemi scelti di analisi matematica. Vol. 1, Liguori.

Semester

First year, First semester.

Assessment method

Written and oral examination (18-30/30).

The written examination evaluates the knowledge of the course contents and the ability to apply them to problem solving. The oral examination requires the exposition of statements and proofs of the theorems, the definitions, the examples / counterexamples and the calculation techniques.

In both examinations, the correctness of the answers, the mathematical language as well as the rigor and clarity of the exposition will be evaluated.

To pass the exam, a score of at least 15 must be obtained in both the practical and theoretical examinations, the arithmetic mean of the two scores must be at least 18. It constitutes the final grade of the exam.

During the year there are 5 exam sessions in the following periods: two in February, one in June, one in July and one in September. Each exam session includes a written examination and then, in case of minimal pass grade, an oral examination within a few days. During the teaching period there are two mid-term written tests which, in case of a positive overall result, will allow to be admitted to the oral examination in February.

Office hours

By appointment.