UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Basic Mathematics

2021-1-G8501R006

Course title

Foundations of mathematics

VERY IMPORTANT

The students of the course Istituzioni di matematiche are subdivided into groups AL (professor Daniela Bertacchi) and MZ (professor Pablo Spiga), according to the initial of the surname. The professor of your group is the one you must refer to for any question.

The professor of your group is the person you must always refer to, addressing any e-mail or communications to Daniela Bertacchi if you are in the AL group, to PabloSpiga if you are in the MZ group.

The enrollment password is
mateal for students AL
matemz for students MZ.

Topics and course structure

This course introduces the basic results in arithmetics and in some other number fields. Moreover, we introduce some elements on sets and on functions and in probability theory.

- Elements of set theory: operations among sets.
- Functions, injective, surjective and bijective functions. Infinite sets.
- Binary relations. Equivalence relations and order relations: equivalence classes and partitions.

- The set of natural numbers. Introduction to the natural numbers via the Peano axioms. Sum, product and order in the natural numbers. Induction principle. Representation of natural numbers in base 10 and in other bases.
- The integers. Construction of the integers from the natural numbers. Divisibility: quotient and remainder. Congruences mod n, and some modular arithmetic. Prime numbers, fundamental theorem of arithmetic. Eratostene's sieve. Greatest common divisor and minimal common multiple via Euclide's algorithm.
- Rational numbers. Construction of the rational numbers from the integers. Elementary properties of rationals: algebraic and topological properties.
- Basic introduction to probability theory. Independent events and conditional probability. Elementary computation of probabilities.A brief introduction to the construction of the real field from the rationals.

Objectives

At the end of the course the student is familiar with the classical mathematical deduction and with some arguments in logic. Moreover, the student is able to present the basic concept in arithmetic from a university level perspective. The student can also deal with elementary probability theory situations.

Methodologies

Lectures 42 hours; during the Covid-19 Emergency the lectures will be subdivided in recorded videos on the fundamental aspects (given in asyncronous mode) and live sessions that will be with a fraction of the students in presence (the live sessions will be recorded and made available on elearning to grant access also to the students which are not in the classroom).

Exercise classes in small groups: 7 encounters for a total of 14 hours (to attend one of these groups, it is compulsory to enrol at the beginning of the semester). Those hours will be remotely taught in asyncronous mode plus eventually live sessions through videoconference. Some exercises will be assigned during every of the seven encounters; students must solve those exercises and deliver them to the teachers. Students who deliver the solutions in the scheduled time will be counted as present during the class.

Exercises on interacting online platform (wims).

Some meetings in small groups to tutor students take place during the year. (Those meetings may be through videoconference due to the Covid-19 emergency).

Online and offline teaching materials

Books (see bibliography).

Online: a list of exercises to be solved on paper, and interactive exercises on the platform wims.

Programme and references for attending students

PROGRAMME

Elements of set theory: operations among sets.

Functions, injective, surjective and bijective functions. Infinite sets.

Binary relations. Equivalence relations and order relations: equivalence classes and partitions.

The set of natural numbers. Introduction to the natural numbers via the Peano axioms. Sum, product and order in the natural numbers. Induction principle. Representation of natural numbers in base 10 and in other bases.

The integers. Construction of the integers from the natural numbers. Divisibility: quotient and remainder. Congruences mod n, and some modular arithmetic. Prime numbers, fundamental theorem of arithmetic. Eratostene's sieve. Greatest common divisor and minimal common multiple via Euclide's algorithm.

Textbook

M. Cazzola, Matematica per scienze della formazione primaria

Further material

- *G. Caiati - A. Castellano, In equilibrio su una linea di numeri, Mimesis, 2007*
- *A. Cerasoli, Io conto, Feltrinelli, 2010*
- *A. Cerasoli, Sono il numero 1, Feltrinelli, 2008*
- *P. Cereda et al, L'aritmetica del Pirata Newton, Mimesis, 2010*
- *P. Cereda – G.Dimitolo, La ciurma del Pirata Newton, Mimesis, 2008*
- *H. M. Enzensberger, Il mago dei numeri, Einaudi*

Programme and references for non-attending students

As for attending students

Assessment methods

- Type of test

The exam consists of two computerized tests (the first denoted as Arithmetic zero and the second denoted Exam test) plus a possible oral test.

During the Covid-19 emergency the exams are remotely assigned. For technical details, see the notices in the course's page.

1. **Aritmetica Zero Test:** it is a test on arithmetic skills that should be acquired from elementary school.

During the test, the use of calculators is not allowed. Registration takes place via the platform http://wims.matapp.unimib.it/ and the test takes place on the same platform (type of test: OPEN QUESTIONS WITH NUMERICAL ANSWERS). This test must be successfully passed (grade greater than or equal to 21/30) on a date prior to that of the written exam (or on the same date as the written exam if a zero arithmetic session is also activated on that day). The test must be passed, the mark is not evaluated in the final mark of the exam. Students are invited to pass it as soon as possible during the academic year (check for the available dates on the wims portal, under "Questionari" where you can also register). Failure to pass Arithmetic zero before the date of the written test involves not admitting to the written exam. The Arithmetic zero test is valid for one academic year. So if passed in the academic year 2020/21 it is valid until 30/9/2021. (The term of validity rule does not apply to those who took the test before 1/10/2015). More information on the test of Arithmetic zero can be found on the site http://wims.matapp.unimib.it/

2. **Exam test:** registration is compulsory on s3w.si.unimib.it. Registration on s3 is also valid for the oral test/registration of the outcome. The exam consists both of exercises similar to those seen in exercise classess and more theoretical questions. The test is computerized on the platform http://wims.matapp.unimib.it/ (where students are invited to practice), plus a part of questions / exercises to be carried out on paper. Type of questions: MULTIPLE CHOICE QUESTIONS and OPEN QUESTIONS WITH NUMERICAL ANSWERS for the part on the wims platform; EXERCISES and/or OPEN QUESTIONS for the part on paper. The duration of the test is approximately 2 hours and during it *the use of textbooks or notes and handouts is NOT allowed*. The use of calculators is not allowed. *During the Covid-19 emergency, the part on paper may take place in different modalities or be suppressed.*

3. **Oral test/verbalization:** The oral exam focuses on the course topics.

   - The oral exam is compulsory for those who obtain a mark between 16 and 20 in the written test (including extremes).
   - Those who obtain a mark in the written test greater than or equal to 26: these students can choose whether to accept a mark of 26 or take an oral test in order to obtain a higher mark (it being understood that the oral test, if unsatisfactory, it can lead to a lowering of the final vote). In practice, if a student obtains 27 in the written test, he can decide to record the mark without taking the oral test: in this case the mark 26 will be recorded. Otherwise, the student can decide to take an oral test: depending on the progress of the test Oral, grade 27 can be lowered, confirmed, or raised.
   - The oral test must also be taken **in all those cases that are requested either by the teacher or by the student**, see also the part related to the partial tests.
   - In all other cases there will be a brief virtual meeting to record the mark, through WebEx or Google Meet.
   - The oral test, where applicable, must be taken in the same session as the written test. In practice, if a student intends to take the written test on the second session of February, then he is also required to take the (possible) oral test in the second session of February and no later. If the oral test does not take place in this appeal, the written test is considered canceled.

4. **Partial tests:** the student of any year of the course can choose to take, in place of the written test, two partial tests (denoted as the first and second "compitino") normally scheduled the first in November and the second coinciding with the first exam session (in January). Partial tests have the same modality as the exam. *To be admitted to the partial tests, attendance of the 7 training sessions is mandatory, in the current academic year*.

5. **Passing the partial tests:** those who obtain in the first partial test a grade greater than or equal to 14 are admitted to the second partial test. The second partial test is passed if a grade greater than or equal to 16 is obtained and the arithmetic mean between the marks of the first and second tests is greater than or equal to 16. If the difference in marks between the two assignments exceeds 8 points, the oral exam is

mandatory. In practice, if for instance a student gets 17 at the first test and 24 at the second, then the written test is considered passed. If a student gets 16 on the first test and 25 on the second test, then the written test is considered passed, but an oral test is compulsorily required since the difference 25-16 = 9 is greater than 8. If a student gets 16 on the first partial test and 20 at the second, then the written test is considered passed, but an oral test is required as the arithmetic average between 16 and 20 is 18 and 18 is between 16 and 20.

6. **Choice between second partial test and first exam:** the date of the first session in January coincides with that of the second partial test. Therefore, those who pass the first assignment can decide whether to take the second partial or the total test on that date. (Check with the notice forum in December if, when and how you must tell the decision).

    0. **Exam sessions.** The exam sessions are 3: one in January-February with three dates, one in June-July with three dates, and one in September with one date.

    0. **Exercise sessions:** rounds of exercise sessions are set up, on a weekly basis, for a total of 7 meetings each. All students interested in attendance are invited to register on the scheduled dates, through the elearning page of the course. Attendance is mandatory for admission to partial tests.

9. **Evaluation criteria:** the correctness of the answers, the completeness and the ability to clearly and precisely argue the course topics are assessed.

Office hours

Office hours by appointment (send an email to your professor):

students AL: Daniela Bertacchi daniela.bertacchi@unimib.it

students MZ: Pablo Spiga pablo.spiga@unimib.it

Programme validity

One year.

Course tutors and assistants

Maurizio Dini

Margherita Montonati

Andrea Proserpio

Paola Riva

Claudio Vailati

Fabio Zucca