UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Mathematical Physics

2021-3-E3501Q059

Aims

Learning the methods for the solution of partial differential equations of Mathematical Physics.

Contents

Introduction to classical partial differential equations of mathematical physics and to the related models: Laplace equation, heat equation and wave equation. Solution methods.

Detailed program

- Introduction to partial differential equations:
    - Maxwell equations, transport equation and Euler equations
- Transport equation
    - characteristics and solution of the initial value problem
- Wave equation
    - Physical models (D'Alembert e Lagrange)
    - Characteristics and casual cone
    - Dependence on the space dimensions: Huygens principle and Kirchhoff solution
    - Lorentz invariance
    - Effects of sources and boundaries (Neumann e Dirichlet)
    - Well-posedness
- Heat equation (Diffusion equation)

    - Physical models (Fick law and probabilistic derivation à la Einstein)
    - Self-similar solutions
    - Fundamental solution and solution of the initial value problem
    - Weak maximum principle
    - Effects of sources and boundaries (Neumann e Dirichlet)
    - Well-posedness
- Comparison between wave and heat equation. Dispersion relation.
    - Hints about the Schroedinger equation
- Laplace equation
    - Radial solutions
    - First and second Green identities
    - Mean property for harmonic functions
    - Strong maximum principle for harmonic functions
    - Dirichlet principle
    - Neumann boundary condition (compatibility conditions) and Dirichlet boundary conditions
    - Poisson equation: representation formula and general solution
    - Green functions
    - Method of images
- Distributions
    - Definition and main properties
    - Dirac delta and Green functions
    - Fourier transform method for computation of propagators
    - Weak solutions
- Burgers-Hopf equation
    - Characteristics and initial value problem.
    - Shocks and their regularization.

Prerequisites

Elements of classical Analysis (I & II). Elements of finite dimensional Euclidean geometry. Elements of Physics (I & II)

Teaching form

Lectures

Until the end of the health emergency, the lessons will be uploaded online in the elearning platform.

Textbook and teaching resource

Textbook:

W. Strauss Partial differential equations, Wiley&Sons

Suggested readings:

S.Salsa, Partial differential equations in action, Springer

L.C. Evans, Partial differential equations, AMS

G. B. Whitham, Linear and nonlinear waves, Wiley&Sons

Semester

2nd semester

Assessment method

Oral exam: solution of exercises, statements and proofs of theorems, relevant examples and physics derivation of equations, solutions of exercises proposed in class.

Five exam sessions (January-February, June, July, September).

Until the end of the current health emergency, the oral exam will be online using WebEx with access from the e-elearning page.

Office hours

By appointment.