

# UNIVERSITÀ DEGLI STUDI DI MILANO-BICOCCA

# SYLLABUS DEL CORSO

# Fisica Matematica

2021-3-E3501Q059

# Aims

Learning the methods for the solution of partial differential equations of Mathematical Physics.

## Contents

Introduction to classical partial differential equations of mathematical physics and to the related models: Laplace equation, heat equation and wave equation. Solution methods.

## **Detailed program**

- Introduction to partial differential equations:
  - Maxwell equations, transport equation and Euler equations
- Transport equation
- characteristics and solution of the initial value problem
- Wave equation
  - Physical models (D'Alembert e Lagrange)
  - · Characteristics and casual cone
  - Dependence on the space dimensions: Huygens principle and Kirchhoff solution
  - Lorentz invariance
  - Effects of sources and boundaries (Neumann e Dirichlet)
  - Well-posedness
- Heat equation (Diffusion equation)

- Physical models (Fick law and probabilistic derivation à la Einstein)
- Self-similar solutions
- Fundamental solution and solution of the initial value problem
- Weak maximum principle
- · Effects of sources and boundaries (Neumann e Dirichlet)
- Well-posedness
- Comparison between wave and heat equation. Dispersion relation.
  - Hints about the Schroedinger equation
- Laplace equation
  - Radial solutions
  - First and second Green identities
  - Mean property for harmonic functions
  - Strong maximum principle for harmonic functions
  - Dirichlet principle
  - · Neumann boundary condition (compatibility conditions) and Dirichlet boundary conditions
  - · Poisson equation: representation formula and general solution
  - Green functions
  - Method of images
- Distributions
  - Definition and main properties
  - · Dirac delta and Green functions
  - · Fourier transform method for computation of propagators
  - Weak solutions
- Burgers-Hopf equation
  - Characteristics and initial value problem.
  - Shocks and their regularization.

#### Prerequisites

Elements of classical Analysis (I & II). Elements of finite dimensional Euclidean geometry. Elements of Physics (I & II)

#### **Teaching form**

Lectures

Until the end of the health emergency, the lessons will be uploaded online in the elearning platform.

#### Textbook and teaching resource

Textbook:

W. Strauss Partial differential equations, Wiley&Sons

Suggested readings:

- S.Salsa, Partial differential equations in action, Springer
- L.C. Evans, Partial differential equations, AMS
- G. B. Whitham, Linear and nonlinear waves, Wiley&Sons

#### Semester

2<sup>nd</sup> semester

#### Assessment method

Oral exam: solution of exercises, statements and proofs of theorems, relevant examples and physics derivation of equations, solutions of exercises proposed in class.

Five exam sessions (January-February, June, July, September).

Until the end of the current health emergency, the oral exam will be online using WebEx with access from the eelearning page.

### **Office hours**

By appointment.