UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Measure Theory

2021-2-E3501Q053

Aims

The students should understand the theoretical aspects and the basic analytic applications of Measure and Intrgration Theory. In particular they should master the Convergence Theorems.

Contents

- The Riemann integral and the problems with passing to the limit.
- Algebras, sigma-algebras and measures. Measurable functions.
- Outer measures, premeasures, extension theorem. Borel and Lebesgue measures.
- Abstract integration. Convergence theorems
- Integration in several variables. Fubini-Tonelli theorem. Change of variables.
- Completeness of L^1.

Detailed program

1. The Riemann integral (an overview) and the problems with passing to the limit. Need of an integral better suited to deal with pointwise convergence of sequences of functions. A possible approach and an obstacle:

Vitali's set.

2. Abstract measure theory. Algebras, sigma-algebras and measures. Basic properties and examples. Complete measures. Borel sigma-algebra. Product of sigma-algebras. Measurable functions. Simple functions. Measurability of the pointwise limit of a sequence of measurable functions. Measurable functions as pointwise limit of simple functions.

3. How to construct relevant measures. Outer measures and a way to generate some of them. Caratheodory condition and theorem. Premeasures and the extension theorem. Borel and Lebesgue measures.

4. Abstract integration. Integration of non negative functions. Monotone convergence theorem, Fatou's Lemma. Integration of complex valued functions. The dominated convergence theorem.

5. Integration in several variables. Fubini-Tonelli theorem. Change of variables.

6. Completeness of L^1.

Prerequisites

The basic courses in Analysis of one and several real variables. A good knowledge of general topology and some abstract algebra are also recommended.

Teaching form

Frontal teaching, possibly in telematic form if the Covid 19 emergency should require it.

Textbook and teaching resource

Notes from the instructor, collections of previous written tests, teaching resource from past years.

Main reference text: Folland, Real Analysis, Wiley

Other texts:

- Ambrosio - Da Prato - Mennucci, Introduction to Measure Theory and Integration, Edizioni della Normale.
- Rudin, Real and Complex Analysis,
- Stein - Shakarchi, Real Analysis, Measure Theory, Integration and Hilbert spaces, Princeton

Semester

Spring semester. March-June 2021.

Assessment method

The exam consists of a written part (exercises) and an oral part about the theory. Passing the written test is mandatory to be admitted to the oral part. During the oral exam there can be a dicussion of the written test..

To pass the exam is required: knowledge and correct use of the convergence theorems; a clear and precise picture of the abstract theory of measure and integration; a good understanding of Borel and Lebesgue measures in one and higher dimensions. The grade will depend on the way the student will be able to state and prove the main theorems.

The written and oral tests concur with equal weight to the grade.

There will be six exam sessions in the academic year: june, july, september, november, february/march, april.

If the Covid 19 emergency will require it, the two part of the exam will be administered in telematic form. The necessary technical instructions will be given to the students shortly before the tests.

Office hours

By appointment, mostly on thursdays 14.00 - 16.00.