UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Algebra II

2021-2-E3501Q010

Aims

On the basis of the knowledge acquired in the Algebra I course, the course is aimed to a) illustrate further topics in the theory of rings and fields; b) develop the theory of finitely generated modules over principal ideal domains, with applications to abelian groups and linear algebra.

Achievements of a successful attendance of the course include

- Knowledge: The knowledge and the understanding of the principle definitions, theorems and results in the theory of rings and their modules, as well as in field theory.
- Capability: The capacity to apply this abstract knowledge to concrete problems in algebra.

Contents

Rings and their modules, and fields

Detailed program

Topics in ring theory. Polynomial extensions. Polynomials in several variables. Noetherian domains. Hilbert's basis theorem.

Localization.

Extensions of rings and fields. Algebraic and transcendental extensions. The splitting field of a polynomial. Finite fields.

Modules over a ring and linear algebra. Free modules: bases. Rank, universal property. Torsion. Modules over principal ideal domains: finitely generated modules; equivalence of matrices and reduction to normal form. Structure theorem for finitely generated modules. Torsion modules and primary decomposition. Invariant factors, elementary divisors. Applications to abelian groups and matrices: Structure theorem for finitely generated abelian groups. Canonical forms of matrices: the companion matrix, rational canonical form, Jordan canonical form.

Prerequisites

Prerequisites: The contents of the courses *Linear algebra and Geometry* and *Algebra I*.

Teaching form

6 credits (ECTS) of lecturing, 2 credits (ECTS) of exercise classes

Textbook and teaching resource

N. Jacobson, Basic Algebra I, Freeman & Co, 1985.

Additional References:

S. Bosch, Algebra, Springer-Verlag, 2003.

B. Hartley & T. Hawkes. Rings, modules and linear algebra, Chapman & Hall 1970

Semester

1st semester

Assessment method

Examination: A **written exam** of ca. 90 minutes (non multiple choice) and an **oral examination** of ca. 20 minutes on the content of the course. Both exams contribute ca. 50 percent to the final mark. Passing the written exam (answering ca. 40 percent of all the questions correctly) is mandatory for being admitted to the oral examination.

In the first call the written exam will be devided into two partial exams. (The first before Christmas, the second after the completion of the course (around first of February)). Students are advised to participate at the first two partial exams in order to practise and to get accustomed to the type of questions they have to answer. Failure in these exams will not have any impact on the final mark.

The questions will concern definitions, examples, counterexamples, exposition and application of Theorems as well as their proofs.

From the second call onward there will be just one written exam covering all the material of the course.

As long as the current health crisis remains, the oral exam will take place remotely using "webex", and the link will be made available through the e-learning page of the course. In case that the participation at a written examination would result in a serious health risk, the exam will be reduced to an oral examination.

Office hours

On appointment