UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Mathematical Analysis II

2021-2-E3501Q008

Aims

A basic course in integral and differential calculus in several variables and ordinary differential equation

Expected learning outcomes include

Knowledge: acquiring the notion of Banach space with some classic examples in mind: continuous / limited functions on a compact inetrval. Understanding of the definitions and main
results of the differential calculus in several variables and of the theory of ordinary differential equations

Capacity: acquire the main integration techniques for functions of several variables in domains delimited by

regular curves as well as the ability to apply the aforementioned abstract knowledge to concrete problems.

Contents

Complete metric spaces and Banach spaces: examples. Sequences and series of functions. Directional derivatives, differentiable functions, higher order derivatives, critical points and local extrema. Integral calculus in several variables: Fubini's theorem; change of variables; polar, spherical and cilindrical coordinates. Ordinary differential equations: existence, uniqueness and continuous dependence on initial data. Implicit function theorem and Lagrange multipliers.

Detailed program

1. Differential calculus in more than one variable.
    1. Directional derivatives. Differentiable functions. Link between the directional derivatives in the case of differentiable functions. Bonds between continuity and derivability and differentiability. Higer order derivatives. Derivatives of compound functions.
    2. Maximum and minimum in open sets: necessary condition for differentiable functions and level curves.
    3. Positive defined/semidefinite matrices and related Matrix criteria Hessian. Taylor's formula arrested on the second order. Convexity and related criteria for the recognition of theirs extrema.
    4. Recognition of the maximums and minima by the Hessian matrix
2. Integral calculus in more than one variable.
    1. Definition of Riemann integral for a real valued function of several variables.
    2. Measurable sets according to Peano-Jordan: necessary and sufficient condition for measurability in the case of bounded sets. Examples of measurable and non-measurable sets.
    3. Reduction method (Fubini's theorem) for multiple integrals. The center of gravity of a two-dimensional measurable set. Solid volume.
    4. Change of variables in double and triple integrals (*): polar, spherical and cylindrical coordinates. Volume of rotation solids: Guldino's theorem.
    5. Improper integrals.
3. Curves and surfaces.
    1. Regular / piecewise regular curves in R^n. Length of a curve: equivalent definitions and independence from the parametrisation (proof of the Theorem 15.4 of the book of Giusti is not required). Curvilinear abscissa
    2. Theorem of implicit functions (Dini) in the two-dimensional case. Cartesian surfaces; sufficient conditions for one surface to be locally a chart. Vector product and theorem of Dini in the case of a function from R^3 to R^2 (without proof)
    3. Maximum and minimum in compact sets: Lagrange multipliers (proof only in the case of a constraint in R^2).
4. Sequences and series of functions
    1. Normed vector spaces: examples (C ([a, b]), B (I) (with I interval) C^n([a, b]). Banach spaces.
    2. Theorem of contractions.
    3. Punctual and uniform convergence for sequences / series of functions. Weierstrass criterion for the series of functions. Uniform convergence and boundedness/ continuity of the limit function. Pointwise / uniform convergence for derivatives of functions. Limits of sequences of integrable functions.
    4. Power series: convergence radius. Taylor series. approximation of integrals through the use of power series.
5. Differential equations
    1. Cauchy problem for first order equations. Theorem of existence and local uniqueness. Extension of solutions. Maximal solution and related properties (*). Sublinearity condition. Theorem of existence and global uniqueness (proof is not required in the sublinear case).
    2. Equations of order n: equivalence with a system of the first order. Linear systems of the first order. Structure of the space of the solutions of a homogeneous system of the first order. Solutions in the non-homogeneous case. The Variation of constants method in the case of an equation of order n.

3. Exponential matrix: definition and properties. Calculation of the exponential matrix in the case where the matrix is ??diagonalizable on the real field. The calculation is required in the other cases only for 3 x 3 matrices.
4. Solution methods for some particular equations and systems.

Prerequisites

Analysis I, Linear Algebra, Geometry I

Teaching form

lessons (8cfu), exercises (4cfu) and tutoring. Lessons in Italian.

Normally, this course is taught by live lessons at the blackboard and live exercise sessions at the blackboard. For all the duration of the current health crisis, lessons will instead take place remotely, by video-recorded lectures that might be either synchronous or asynchronous, and will be made available to the students on the moodle platform. Regarding the exercise sessions of this course, compatibly with the constraints imposed by the availability of classrooms and the bound on total student presence on campus, some activities in presence are foreseen.

Textbook and teaching resource

Textbook: C.Pagani; S.Salsa: Analisi Matematica 2 Ed. Zanichelli

Other teaching resources

Enrico Giusti: Analisi Matematica II ed. Bollati Boringhieri.

A. Bacciotti; F. Ricci: Lezioni di Analisi Matematica 2 Ed. Levrotto & Bella /Torino

C.Pagani; S.Salsa: Analisi Matematica 1 Ed. Zanichelli

Enrico Giusti: Analisi Matematica 2, old edition, Bollati Boringhieri

Semester

First semester.

Assessment method

In the Covid-19 emergency period, the oral exams will only be online. They will be carried out using the WebEx platform: a public link will be provided on the e-learning page of the course for access to the examination of possible virtual spectators.

For the written tests, always carried out remotely, we refer to the methods described on the course website.

Oral and written exam: weighted 1/2 and 1/2 of the final mark

In the written test it is required to demonstrate to be able to apply the theoretical contents of the course to solve problems. The oral test requires the ability to expose the statements and proofs of the theorems, the definitions, the examples / counterexamples and the calculation techniques introduced. See the file 'Exams' for more technicalities.

During the year there are 5 exam sessions in the following periods: two in the month of February, one in June, one in July and one in September. Each exam session includes a written test and then, in case of passing the written test, a theoretical / oral test a few days away. If possible, depending on the covid-19 restrictions, during the period of the lessons there will be two partial written tests, reserved to second year students. Those which obtain a positive overall result, will be allowed to directly support the oral exam in the month of February.

Minimum grade in the written part: 16/33 to be admitted to the oral exam

Office hours

By appointment, by Webex