UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Approssimazione di Equazioni Differenziali Ordinarie

2021-1-F4001Q105

Aims

The main goals of the course are:

- Knowledge (and understanding) of the different numerical methods for ODEs
- Ability to construct and analyse numerical methods for approximating systems of ordinary differential equations
- Knowledge (and understanding) of some of the techniques **of data assimilation.**
- Ability to choose the appropriate numerical method for concrete problems
- Ability to interpret and analyse numerical results
- Ability to implement the resulting numerical algorithms efficiently

Contents

This course is concerned with the development and analysis of numerical methods for differential equations. Topics covered include: well-posedness of initial value problems, analysis of Euler's method, Runge-Kutta methods, methods for stiff problems and Geometric numerical integration approaches.

In the last part of the course we will introduce some of basic techniques in data assimilation. In particular the problems of smoothing and filtering will be discussed together with their basic algorithms:Markov Chain Monte Carlo e Metropoli-Hastings (smoothing); Kalman Filter; Ensemble Kalman Filter.

We shall cover the subject from mathematical point of view, studying how to construct modern computational algorithms, exploring their properties and validating the algorithms in concrete problems.

Detailed program

0- Introduction.

Recap of the theory of ordinary differential equations and systems of ODEs. Well posedness results. Recap on numerical integration (numerical quadrature)

1. -One-step schemes:

Euler method. Convergence theory. Explicit Runge-Kutta (RK) methods. Convergence Theory. Hint on Order conditions . Richardson extrapolation. Embedded Runge-Kutta methods.

2.- Collocation methods(I).

Recap on Gaussian Quadrature. Construction of collocation methods. Convergence analysis of implicit RK.

3.- Linear Stability and Stiffness.

Linear Stability lineare. Stability of RK methods. Stiff problems.
BDF method (Backward Differential Formula).

4.- Collocation methods (II). Implementation of implicit RK. Partitioned methods and Splitting methods: Trotter-Lie and Strang splittings.

5.- Geometric integrators.

Hamiltonian Systems. Numerical conservation of invariants. Symmetric integrators. Symplectic integrators. Stormer-Verlet method.

6- Data Assimilation:

Recap of some basic probability theory. Monte Carlo and Importance Sampling. Markov Chain Montecarlo. Metropolis-Hastings algorithm. Filtering and Smoothing problems. Kalman Filter and generalizations.

Prerequisites

Solid knowledge of Analysis, Linear Algebra and basic Numerical Analysis.
Solid knowledge of Ordinary differential equations and basic knowledge of MATLAB

Teaching form

Lectures in class and in the Lab.

We will use MATLAB for all computer examples, exercises and projects.

The students will be given the possibility of adhering to the use of "flipped classroom" (or inverse-blended teaching)for a few of the topics of the course. This option will be completely optional.

During the Covid-19 emergency period *the classes will be delivered remotely, making use of video recording. Some special events and clarifications/tutorials will be delivered by direct online meetings. The computer lab classes will be delivered in a mixed way, partially by video recording and partially by direct online meetings.*

Textbook and teaching resource

Different material will be provided during the course. The course has a big practical component for which we will use MATLAB

We will use several books(several chapters in each of them to cover the different topics)

Bibliography:

-E. Hairer and S. P. Norsett and G. Wanner, “Solving Ordinary Differential Equations I ”, Springer, Berlin, 1993.
-E. Hairer and G. Wanner, “Solving Ordinary Differential Equations II ”, Springer, Berlin, 1996.
-E. Hairer, C. Lubich and G. Wanner, “Geometric Numerical Integration”, second edition, Springer, Berlin, 2006.
-B. Leimkuhler and S. Reich, “Simulating Hamiltonian Dynamics”, Cambridge University
Press, 2005.
-K .J. H. Law, A. M. Stuart and K. C. Zygalakis, *Data Assimilation: A Mathematical Introduction.* Springer, (2015)

Semester

First semester

Assessment method

The evaluation of the course has two parts:

1- the development of a small project
2- a small (oral or written) exam. Specifics on the oral or written exam will be given later on during the course.

The students who adhere to the flipped classroom will have the extra vote from their exposition.

The small project could be chosen from a list of projects that will be made available to the students towards the end of the course. Students are encouraged to work on the project **in groups** of at most two or three people. The project should be handed four days before the before the date of the small exam. Part of the small exam will be devoted to the discussion of the project, allowing to validate the knowledge and capabilities of the students related to the course.

Office hours

By appointment (that should be fixed by writing an email to me)