UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

# COURSE SYLLABUS

## Differential Geometry

**2021-1-F4001Q071**

### Aims

The aim of the course is to introduce the student to the theory of Riemannian manifolds, namely differentiable manifolds endowed with a Riemannian metric, consisting of a Euclidean scalar product assigned to each tangent space, depending smoothly on the base point. The course aims at making the student familiar with the basic tools of differential geometry, starting from the essential notion of connection - the generalization to abstract manifolds of the derivative of a vector field - along with the related concepts of geodesics and curvature. The language of Lie groups will then be introduced, enabling one to describe the symmetries of a Riemannian metric at the infinitesimal (holonomy group) and global level (isometry group).

The expected learning outcomes include the following:

- the knowledge and understanding of the basic definitions and statements, as well as the basic strategies of proof in differential geometry; the knowledge and understanding of some of the key examples in which the theory manifests itself;
- the ability to apply the acquired abstract knowledge to the solution of simple computational exercises and theoretical problems, referring in a precise and well-organized manner to the pertinent results; the ability to apply the theoretical background to the construction and discussion of simple examples and solution of simple exercises; the ability to expose and communicate effectively and clearly the theoretical content of the course.

### Contents

Riemannian metrics and distance, connections and curvature, geodesics and exponential map, Lie groups and algebras, invariant metrics, isometry group and holonomy group.

## Detailed program

Vector bundles. Connections, torsion, geodesics, exponential map. Riemannian manifolds, Levi-Civita connection. Curvature of a connection; Bianchi identities; sectional, Ricci and scalar curvature. Curvature of a submanifold; Teorema Egregium. Lie algebras; adjoint representation. Lie groups; Lie group - Lie algebra correspondence. Invariant metrics on a Lie group. Holonomy group; Ambrose-Singer theorem, De Rham theorem. Killing fields and isometry group.

## Prerequisites

Differential calculus in several variables, basic notions of differentiable manifolds, linear and multilinear algebra.

## Teaching form

Lectures (8 CFU)

Pending the current health emergency, lectures will be held remotely, in the form of recorded lectures, either synchronous or asynchronous, which will be made available to the students on the e-learning webpage. In order to improve student involvement, online lectures will be integrated with synchronous events which may be held remotely by videoconference, or, if appropriate, in the lecture room, with students divided in groups.

## Textbook and teaching resource

J. Lee. Introduction to Riemannian manifolds. Springer.

P. Petersen, Riemannian geometry. Springer.

Lecture notes on the e-learning webpage.

## Semester

Second semester

## Assessment method

Oral exam

The oral exam will consist in a short exercise and some theoretical questions. The exercise will consist in an

explicit computation of one of the objects introduced during the course (for instance, the curvature of a given metric). Theoretical questions will cover definitions, theorem statements, proofs, construction of examples or counterexamples and simple theoretical problems.

Pending the current health emergency, the exam will be held remotely through WebEx or similar, with access made available on the e-learning webpage of the course.

## Office hours

By appointment. Due to the current health emergency, student reception will be carried out online through WebEx or analogous.