UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Functional Analysis

2021-1-F4001Q075

Aims

Consistent with the educational objectives of the Master's Degree in Mathematics, the course aims to provide students with the knowledge concerning the definitions and the basic statements of the Functional Analysis. The skills needed to understand and analyze the main techniques and demonstration methods related to the theory, and the skills to apply them to face problems to different areas of Mathematics will be also provided. Particular emphasis will be placed on topological aspects.

Contents

Locally compact Hausdorff spaces. Spaces of continuous functions. Spaces L^p. Compactness in L^p and in C^0. Weak and weak* (weak star) topology. Riesz representation theorems.

Detailed program

Metric spaces, normed vector spaces and compactness.

Linear functionals and weak topology on a normed space. Sub-additive positively homogenous functionals. The Hahn-Banach theorem: general form. Convexity and hyperplane separation. Mazur Theorem: weak and strong closure of convex sets.

Reflexive spaces. Reflexivity in the L^p spaces. Uniform convexity and reflexivity. Weak and strong convergence in uniformly convex spaces. Kakutani and Eberlein-Smulyan theorems: weak compactness of the closed ball and reflexivity. Sequential compactness in the weak* topology.

Locally convex topological vector spaces. Convex hull and extremal points: the Krein-Milman theorem.

Prerequisites

Elements of the theory of abstract integration, elements of L^p space theory , elements of general topology. Basic knowledge of Banach spaces and Hilbert spaces.

Teaching form

Until the end of the Covid-19 emergency, lectures will be held in remote mode. All lectures will be recorded and made available on the E-Learning site of the teaching. In order to help and improve the students involvement, special sections of discussion will be scheduled. These will take place either in remote mode or in a University room: in this latter case, students will be divided in small groups if needed.

Textbook and teaching resource

Bibliographic references

H. Brezis. *Functional analysis, Sobolev spaces and partial differential equations*. Universitext. Springer, New York, 2011
H. L. Royden. *Real analysis*. Macmillan Publishing Company, New York, third edition, 1988
W. Rudin. *Real and complex analysis*. McGraw-Hill Book Co., New York, third edition, 1987
T. B•uhler and D. A. Salamon. *Functional analysis.* volume 191 of Graduate Studies in Mathematics. AMS, Providence, RI, 2018.
Further material will be shared on the E-Learning site of the teaching.

Semester

First semester.

Assessment method

The exam is solely oral and consists of a colloquium with assessment. It is divided into a series of questions designed to verify the student's knowledge and mastery of the theorems with related demonstrations carried out

during the course.

In the oral exam it is assessed whether the student has acquired the necessary skills to present a selection of the demonstrations carried out in the classroom, and, above all, the critical and operational knowledge of the definitions and results of the course, also by illustrating examples and counter-examples.

Until the end of the Covid-19 emergency, oral exams will be held on the WebEx videoconferencing platform (or similar). The link to the virtual room will be made available on the E-Learning site of the teaching.

Office hours

By appointment