UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Real Analysis and Differential Equations

2021-1-F4001Q076

Aims

According to the Mathematics Degree educational objectives, the course aim is the introduction to linear partial differential equations with hints to nonlinear ones. The skills needed to understand and analyse the most important techniques in the theory and the ability to solve exercises and problems will be provided.

Contents

Spectral theory for compact and selfadjoint operators. Elliptic equations: regularity, maximum principles, eigenvalues and eigenfunctions of the Laplacian. Bochner integral. Linear parabolic and hyperbolic partial differential equations. Hyperbolic systems of first order equations.

Detailed program

Spectral theory: Definitions of adjoint, selfadjoint, compact operators, spectrum. Properties. Spectrum of compact operators. Spectral decomposition of selfadjoint compact operators. Fredholm alternative.
Second order elliptic equations: elliptic operators, classical and weak solutions, regularity of weak solutions, maximum principles, eigenvalues and eigenfunctions of the Laplacian.
Bochner integral: Definition, main properties and Sobolev spaces defined with the Bochner integral.
Parabolic equations: Fundamental solution for the heat equation and its application. Duhamel's principle. Weak solutions for second order parabolic equations. Energy estimates, existence and uniqueness of weak solutions. Regularity. Maximum principle. The Banach fixed point applied to the study of Semilinear parabolic equations**.**
Linear hyperbolic equations: The method of characteristics applied to the transport equation and to hyperbolic linear systems of differential equations with constant coefficients.

Prerequisites

Main results in functional analysis, bounded linear operators in Banach spaces, weak topologies, spaces of continuous and H\"older continuous functions, $\mathbf{L}^p$ spaces, their duals and properties, Sobolev spaces and immersion theorems.

Teaching form

Lectures in classroom where definitions, results and relevant examples are illustrated (sometimes with relation to extra-mathematical applications as well).

Course delivered in Italian with the possibility of being delivered in English if foreign students request it.

Textbook and teaching resource

- A. Bressan. Hyperbolic systems of conservation laws: the one-dimensional Cauchy problem. Vol. 20. Oxford University Press on Demand, 2000.
- A. Bressan. Lecture Notes on Functional Analysis. With applications to linear partial differential equations. American Mathematical Society, 2013.
- H. Brezis. Functional analysis, Sobolev spaces and partial differential equations. Springer Science and Business Media, 2010.
- L. C. Evans, Partial Differential Equations, AMS Graduate Studies in Mathematics, Vol.19. Second Edition, Providence 2010.
- D. Gilbarg, N. S. Trudinger, Elliptic partial differential equations of second order, Reprint of the 1998 edition. Classics in Mathematics. Springer-Verlag, Berlin, 2001.

Course webpage: https://elearning.unimib.it/course/view.php?id=25417

Semester

Second semester.

Assessment method

Written examination. Mark out of thirty. The student is asked to develop two topics out of three proposed at the examination in two hours. The written discussion must be precise, detailed, comprehensive and consistent with the proposed topic. Moreover it must contain some of the most significant proofs. The ability to present a selection of proofs and, above all, the critical and operational knowledge of the definitions and results presented during the course is evaluated, also by the illustration of examples and counterexamples.

During Covid-19 emergency period, the written examination will be telematic. It will be followed by an oral session conducted through the WebEx platform. A public link to allow access to the examination of possible virtual spectators will be published on the e-learning page of the course.

Office hours

By appointment.