UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Metodi della Fisica Matematica

2021-1-F4001Q063

Aims

The course presents the physical and mathematical foundations of classical field theory. Its leitmotiv is the theory of wave motion: linear and nonlinear waves, dispersive and nondispersive waves. The first part of the course will discuss the general principles of continuum mechanics, and especially of Fluid Dynamics, some applications and examples of which will be dealt with in details. Wave motion is introduced with sound waves (highlighting the analogy with electromagnetic waves) and water waves. The concepts of phase and group velocity, wave packets, geometric "optics" and eikonal equations will be discussed. Then, an important chapter will be devoted to the Schrödinger equation, on the fundamental concepts of Quantum Mechanics and their mathematical implementation. The last part of the course is devoted to the study of a class of non-linear equations, known as soliton equations of the Korteweg-de Vries type.

The main expected learning outcomes are:

1) The knowledge and understanding of the definitions of fluid mechanics, wave theory and the Schrödinger equation; the knowledge of the physical motivations thereof, of the main theoretical results and of the basic strategies for their proofs.

2) The mastering of the different approximations needed in the modelling processes (such as constitutive equations, linearization processes, asymptotic expansions) discussed during the course.

3) The ability to apply such a conceptual background in the analysis of the various applications; the acquisition/improvement of the skill in presenting and clearly discussing both the theoretical contents of the matter and their implementation in specific situations, possibly related with a broader scientific area.

4) The skill to build on the acquired knowledge by further refinements to be used in the analysis of subjects not fully developed during the lectures.

Contents

Continuum mechanics and field theory. Foundations of Fluid Dynamics. Linear and non-linear waves, dispersion. Geometrical optics. Schrödinger equation and wave mechanics. Shallow water waves and equations of the Korteweg - de Vries type.

Detailed program

Contnuous bodies and their configuration space. Fluids and the velocity field.

Euler and Navier-Stokes equations.

Bernoulli theorem, flows past a body, and aerofoil theory.

Sound waves in gas dynamics. Electromagnetic waves in vacuum. Water gravity waves.

Phase and group speed, wave packets, geometric optics and eikonal equation.

The Hamilton jacobi equation as the eikonal equation to the Schrödinger equation.

Foundations of Quantum Mechanics and their mathematical formulation.

Shallow water waves: the Korteweg de Vries equation. Properties and methods of solution.

Prerequisites

No course of the Master Degree in Mathematics is strictly required for attending the present course. The basic notions of the courses Mathematical Analysis I and II, Linear algebra and Geometry, Physics I and II and Dynamical Systems and Classical Mechanics of the Batchelor Degree are needed. The prior knowledge of the contents of the courses Complex Analysis and Mathematical Physics of the third year of the Batchelor Degree are advisable.

Teaching form

Lectures (8CFU).

During the COVID-19 emergency, videos of the lectures will be posted on-line. Some events will take place via videoconference meetings.

Textbook and teaching resource

Reference texts:

Selected chapters from:

1. G.B. Whitham: Linear and Nonlinear waves, John Wiley & Sons, 1999.

2. G. Falkovich, Fluid Mechanics (a short course for physicists). Cambridge University Press, 2011.
3. S. Salsa: Partial Differential Equations in Action: from Modeling to theory. Springer, 2008.
4. L.A Takhtajan, Quantum Mechanics for Mathematicians, Springer, 2008.

The notes of the lectures will be regularly published on the e-learning page of the course.

Semester

First semester.

Assessment method

The examination is oral, and consists in two parts.

The first part is the presentation of a written homework on a subject chosen within a list provided by the end of the lectures by the instructor. The list will comprise (also) items complementary to those discussed in the lectures. The student should inform the instructor about her/his choice of the subject of the homework at least 10 days befor the discussion date. Also, she/he must send a copy of the homework to the instructor at least 2 days before that date for a preliminary evaluation.

The main aim of this first part mainly regards points 3 and 4 of the above-mentioned "expected learning outcomes". The evaluation will be based the clarity of the exposition, the ability to synthetize the subject as well as the degree of mastering of the subject acquired by the student. In these respects, the complexity of the chosen homework subject will be also taken into account.

In the second part of the examination, the student will be asked to discuss a few of the main points of the program (at the instructor's choice). This part mainly addresses points 1 and 2 of the "expected learning outcomes".

For what the mark is concerned, the relative weight of the two parts is equal.

Until the end of the COVID-19 emergency, the oral examination will be held via a videoconference webex meeting to be hosted from the e-learning page of this course.

Office hours

Meetings whose schedule is to be agreed either via e-mail (preferred) or this e-learning page. According to the evolution of the COVID-19 emergency and/or of the mutual opportunity, such meetings might be held in videoconference.