UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Numerical Methods For Partial Differential Equations

2021-1-F4001Q103

Aims

In line with the educational objectives of the Master Degree in Mathematics, the course aims to provide the knowledge of the rigorous mathematical theory of the **Finite Element Method** for the approximation of linear elliptic second-order partial differential equations.

At the end of the course the students will have the skills needed to understand more advanced aspects of the method, both with individual work and with other courses.

The method will be implemented in MATLAB, and with the developed codes the students will have the ability to solve simple real-life problems connected with the approximation of partial differential equations.

Contents

- Sobolev Spaces
- Lax-Milgram Lemma
- Galerkin methods
- Cea's Lemma
- Linear Finite Elements
- Lagrange Finite Elements of order k
- error estimates in the energy norm
- Bramble-Hilbert Lemma
- Aubin-Nitsche duality argument

Detailed program

- **Basic concepts.** Presentation in the one-dimensional case of the techniques and the ideas which will be studied in the rest of the course.
- **Sobolev Spaces.** The natural functional environment for the mathematical analysis of the finite element method.
- **Variational Formulation of Elliptic Boundary Value Problems.** Abstract setting for the partial differential equations which will be studied in the course.
- **The Construction of a Finite Element Space.** How to build a finite element.
- **Polynomial Approximation Theory in Sobolev Spaces.** The core of the course. We will study how finite elements (in essence, continuous, piecewise smooth functions) approximate functions in Sobolev Spaces.
- **n-Dimensional Variational Problems.** Examples of partial differential equations which can be approximated with the finite element method.

Prerequisites

Courses of the Laurea Triennale. It is recommended the course Analisi Funzionale of the 1st semester.

Teaching form

Lessons (6 CFU), exercise classes with blackboard and computer (2 CFU).

Due to the COVID-19 pandemic, lessons will be recorded and, if possibile, live-streamed. Exercise classes will be live-streamed and recorded.

Textbook and teaching resource

The reference text is S. C. Brenner e L. R. Scott: The Mathematical Theory of Finite Element Methods, Springer 2008. Teacher's notes on specific topics will also be available.

Semester

2nd semester

Assessment method

The final examination is split into two parts:

- writing and presenting a project;
- oral examitation.

Mark is out of thirty.

The project consists in implementing the approximation of a problem related to partial differential equations, using the codes developed during the course. Tha aim is to test the ability to use the developed instruments. Group working is encouraged (max 3 students) and the quality of the exposition will be part of the mark.

The oral examination will evaluate the knowledge of the definitions, results and rigourous proofs developed in the course; the capacity to understand what are the key points of the theory will also be checke.

There will be 5 exam sessions (in June, July, September, January, February).

Due to the COVID-19 pandemic, the whole examination will be conducted online.

Office hours

On appointment (online via Webex).