SYLLABUS DEL CORSO

Matematica per l'Economia M

2021-1-F8204B021

Learning objectives

Contents

The contents consist of three parts:

In the First Part, basic elements of the theory of ordinary differential equation systems are provided.

In the Second Part, an approach to (continuous time) optimal control problems is presented, along with

a solution existence result.

In the Third Part, basic elements of the measure theory and of the integration theory are provided. As a special case, the Lebesgue integral is considered and convergence theorems are discussed.

Detailed program

Part I (ODE):

- Differential equations in mathematical economics, Cauchy problems and related solution notion.

- Reduction to first order ODE of higher order ODE.

- Solving explicitly classes of differential equations: separable equations, linear equations, Bernoulli's equation.

- Application to specific models (market price dynamics, Solow model of economic growth).

- Global and local solution existence and uniqueness for a Cauchy problem.

- Equilibria and their stability (in the Lyapunov sense, local and global asymptotic).

- Linear ODE systems: solution methods and stability.

Part II (Optimal control):

- Problem statement.

- The Pontryagin maximum principle (the linear dynamics case and beyond).

- Sufficient optimality conditions (Mangasarian condition and Arrow condition).

- Appllications to economical models (optimal selling strategies, selling maximization).

- The simplest problem of the calculus of variations as a special optimal control problem and its application (optimal consumption model).
- A glimpse of sensitivity and shadow prices.

- Existence of an optimal control (Filippov's theorem).

Part III (Selected topics in measure theory):

- Algebra and ?-algebra, generated ?-algebra.

- Measures and their properties.

- The Lebesgue measure on $\mathbf{R}^n$.

- Measurable functions and their properties.

- Integral over a measure space and its properties.

- Integral functions and absolutely continuous functions.

- Convergence theorems (Lebesgue's dominated convergence theorem and monotone convergence theorem).

- Riemann vs Lebesgue integral.

Prerequisites

No prerequisite. A refreshement concerning the following topics in Mathematics

is strongly advised:

- Basic notions about complex numbers;

- Multivariable calculus;

- Matrix calculus with basic elements of linear algebra;

- Eigenvalues and matrix diagonalization methods;

- Quadratic forms;

- Convexity for sets and functions.

Teaching methods

Class lectures.

During the teaching period, some exercise sessions are organized.

Assessment methods

Students are supposed to pass a written exminaton. There are no interim assessment.

A written examination consists of the following kind of questions:

- problem solving;

- discussion in details of models contained in the examination program;

- discussion of some theoretical topics and application of them to specific cases.

Material for exam simulations is also provided.

Textbooks and Reading Materials

Lecture notes and exercises are provided during the course.

Some further reading:

1. **A. Guerraggio - S. Salsa,** *Metodi matematici per l'economia e le scienze sociali*, **G. Giappichelli Editore, Torino, 1997.**

2. **K. Sydsæter - P. Hammond - A. Seierstad - A. Strøm,** *Further Mathematics for Economic Analysis*, **Prentice Hall, Harlow, 2008.**

Semester

The course is scheduled in the second half of the second semester.

Teaching language

Italian.