UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Calculus II

2122-2-E3201Q040

Aims

- To acquire a basic knowledge of complex numbers theory, linear algebra, differential calculus in several real variables and differential equations.
- To acquire the ability to apply the learned methods and techniques both to the solution of exercises and abstract problems and to the mathematical modeling of problems related to the experimental sciences within the scope of the Degree Course.
- To acquire the ability of independently make judgments in the application of the learned methodologies to the modeling and solving of environmental problems.
- To acquire the ability to present in a precise and exhaustive way both the learned theoretical knowledge and the independently developed solutions of exercises and problems.
- To be able to understand the modeling/mathematical contents of the courses delivered within the Degree Course.

Contents

- Complex numbers.
- Vectors in $\mathbf{R}^n$, matrices and linear systems of equations.
- Differentiation and integration in $\mathbf{R}^n$.
- Differential equations**.**

Detailed program

Complex numbers

Definition. Modulus, argument, complex conjugate and their properties. Algebraic and trigonometric form. Power and nth root of a complex number, fundamental theorem of algebra, Euler's identity.

Linear algebra

Vector spaces: sum of vectors, product for a scalar. The vector space $\mathbf{R}^n$: inner product, norm of a vector and its properties. Schwarz inequality, triangular inequality, linear combinations, dependent and independent vectors. Bases and their properties, dimension of a vector space. Matrices and matrix operations: transposed matrix, sum of matrices, product for a scalar and product between matrices. Linear functions. Square matrices: identity matrix, determinant, singular matrices and inverse matrix. Rank of a matrix, its meaning and its relation to the independence of a set of vectors. Systems of linear equations: Gauss elimination method and Rouché-Capelli theorem. Eigenvalues ??and eigenvectors of square matrices and their determination. Symmetric matrices and spectral theorem. Quadratic forms and sign of a quadratic form. Determination of the sign of a quadratic form in *n* variables and the special case of 2 variables. Relationship between the sign of a quadratic form and the sign of the eigenvalues ??of the associated matrix. Relationship between definite quadratic forms and the norm of the vector space.

Curves

Vector functions of a real variable, limits and continuity. Curves, closed curves, simple curves and plane curves. Support of a curve. Derivative and tangent vector to a curve. Regular and piecewise regular curves. Plane curves in polar form. Length of a C^1 curve. Length of graphs of real functions of real variable. Integrals of vector valued functions. The module of the integral is less than or equal to the integral of the module. Line integrals.

Differential calculus for functions of several real variables

Sets in $\mathbf{R}^n$. Spherical neighborhoods. Internal, external and boundary points. Open sets: examples and properties. Closed sets: examples and properties. Bounded sets. Functions of several real variables: introduction and first examples, state functions of thermodinamics. Graphs and level sets. Definition and properties of limits for functions of several variables. Infinite limit to the finite and finite limit to infinity. Continuous functions and their properties. Weierstrass theorem. Partial derivatives and gradient, definition of differentiability, link between differentiability and continuity and between differentiability and derivability. Derivability along a given direction and the gradient formula, geometric meaning of the gradient. Sufficient condition for the differentiability and the class $C^1(\mathbf{R}^n, \mathbf{R})$. The first differential. Derivative of the composite function: the case $p(\underline{x}) = g(f(\underline{x}))$ with $f: \mathbf{R}^n \to \mathbf{R}$ and $g: \mathbf{R} \to \mathbf{R}$ and the case $p(t) = f(\underline{r}(t))$ with $f: \mathbf{R}^n \to \mathbf{R}$ and $\underline{r}: \mathbf{R} \to \mathbf{R}^n$. Level curves and the gradient. Positively homogeneous functions and Euler theorem, application to thermodynamic potentials. Mean value theorem. Higher order derivatives and the Hessian matrix. Schwarz's theorem and the C^2 class. Maxwell's relations in thermodynamics. Taylor's formula Lagrange and Peano remainders. Vector functions of several real variables, Jacobian matrix. General case of the derivation theorem for the composite function.

Optimization

Extreme points. Free and constrained extrema. Stationary (or critical) points. Necessary condition for extrema (Fermat's theorem). Quadratic form associated with the Hessian matrix and its link with the nature of the critical points. Least squares linear approximation.

Multiple integrals

Geometric meaning and properties of the double integral. Simple domains. Computation of double integrals through reduction on rectangular domains and on simple domains. Change of variables. General formula for the change of variables in double integrals. Polar coordinates in double integrals.

Differential equations

Definition. Ordinary and partial differential equations with examples. Exponential growth model and logistic model. Order of a differential equation and systems of differential equations. Differential equations in normal form and equivalence with first order systems. Cauchy problem. Cauchy problem for differential equations in normal form of order n. Existence theorem (Peano) and local existence and uniqueness theorem. Differential equations with separable variables and linear differential equations of the first order. Homogeneous and non homogeneous linear differential equations of order n. General structure of the solution. Linear homogeneous equations with constant coefficients. Differential equations associated to the *RLC* circuit and to the damped harmonic oscillator and their solution. Particular solution of non homogeneous linear equations with constant coefficients when the non-homogeneous term is a polynomial or a real / complex exponential (similarity method). Solution of the equation associated with the *RLC* circuit with periodic forcing. Introduction to the qualitative solution of autonomous differential equations: single equations and 2X2 systems. Qualitative analysis of the solutions of the following models: logistic equation; logistic equation with extinction and collection; Predator prey Lotka-Volterra model; two competing species model.

Prerequisites

Differential and integral calculus for real functions of a single real variable. Even if it is not formally required, it is necessary to know and to be able to handle the contents of Mathematics I in order to be able to follow the course profitably.

Teaching form

Lessons (delivered in Italian), 6 ECTS (48 hours)

Exercise classes, 2 ECTS (20 hours)

Textbook and teaching resource

Matematica Generale, A. Guerraggio, Bollati Boringhieri. (Complex numbers and linear algebra)

Analisi Matematica II, M. Bramanti, C.D. Pagani, S. Salsa, ZANICHELLI. (Differential calculus for functions of several real variables and differential equations)

Esercitazioni di Analisi Matematica 2, M. Bramanti, Esculapio, Bologna. (Exercises)

https://elearning.unimib.it/course/view.php?id=24206

Semester

First semester

Assessment method

The exam is structured in two halves.

1) **Practical half** (written): It evaluates the knowledge of the course contents and the ability to apply them to problem solving. The student is requested to solve some exercises, usually 6 in 120 minutes. Each exercise gives 5 points unless otherwise specified. To be admitted to the theoretical half it is necessary to obtain a score of at least 16.
2) **Theoretical half**:
It consists of two parts. The first part consists of 4 open questions (written). The second part consists of an oral discussion of the first part and on all the topics covered in the course. The ability to clearly present the course topics, the definitions, the statements and some proofs of the theorems are evaluated. To pass the exam, a score of at least 16 must be obtained in each of the two halves, in addition the arithmetic average of the two scores must be at least 18. This average is the final grade given to the exam.

•Two tests are normally held during the teaching period. They replace the practical part if a minimum score of 14 is attained in each test. The tests consist of 10 closed ended questions each with the following score: correct answer 3 points, wrong answer -1 points, no answer 0 points.

Office hours

By appointment