UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Algebra Lineare

2122-1-E4101B002

Learning objectives

The course plans to introduce basic results in linear algebra required for other courses.

Contents

1. Complex Numbers and the fundamental Theorem of Algebra
2. Vector Spaces
3. Inner product spaces
4. Linear Maps
5. Matrices and their operations
6. Systems of linear equations
7. Spectrum of a linear map
8. Diagonalization of matrices and of linear maps
9. Quadratic Forms and signature

Detailed program

Sum and product by scalars for vectors in R ^ n and between matrices. Multiplication between matrices. Systems of linear equations, number of solutions, Gauss resolution method, complete and incomplete matrix associated to a linear system. Homogeneous linear systems and non-homogeneous linear systems. Abstract definition of vector space. Definition of vector subspace. Intersection of vector subspaces and sum of vector subspaces. Linearly dependent and linearly independent vectors. Generator Vectors. Basic definition and then dimension of a vector space. Fundamental theorems on bases, generators vectors and linearly independent vectors. Definition of vector subspace generated by a set of vectors and explicit description in case the set of vectors is finished. Determinant of a matrix. Definition of linear application and its basic properties. Core and image of a linear function, nullity theorem + rank. Matrix associated with linear applications, examples, and basic properties of the matrix associated with linear applications. Discussion between the nucleus of a linear application and the nucleus of the matrix associated to the linear application. Discussion between the image of a linear application and the span of the

columns of the matrix associated with the linear application. Discussion of the relationship between the rank of a matrix and the size of the span of the rows, and the size of the span of the columns. Fundamental change formula for a linear application (only in the case where the domain coincides with the codomain). Gauss method for the calculation of the inverse matrix. Endomorphisms. Eigenvalues and eigenvectors for endomorphisms. Spectrum of matrices and linear applications. Diagonalization of matrices and linear applications. Classification of quadratic forms. Elements on complex numbers.

Prerequisites

No prerequisites are required.

Teaching methods

Frontal Lectures. Exercise session about theoretical contents. Online Tutoring activity for computer-based exercises.

Assessment methods

The exam consists of a computerized test plus a possible oral exam. The exam consists of exercises similar to those seen in exercises, and questions with more theoretical content. The test is computerized, but the answer to some types of questions / exercises can also be requested on paper. The duration of the test is 2 hours and during the use of textbooks or notes and handouts is not allowed. The use of calculators is not allowed.

The oral part of the exam is **not** mandatory for everyone.
- Students whose grade for the written exam is 16 or less will have to take the exam again at a later time.
-The oral part of the exam is *mandatory* for everyone whose grade for the written exam is either 17, 18 or 19.
-The oral part of the exam is *elective* for those attaining a grade greater or equal to 27 in the written exam: such students can choose to either accept a final grade of 27 or take the oral part of the exam to achieve an even better grade (it being understood that if the oral part of the exam is unsatisfactory it can lead to a worsening of the grade and even to a failure of the whole exam). To be clear: a student achieving a 28 in the written part of the exam can choose to register a final grade of 27 (and not take the oral part of the exam) or to take the oral part of the exam: depending on the oral part of the exam the 28 can either be lowered (even to a possible failure of the exam), confirmed or improved.
- To those whose grade for the written part of the test is at least 17: the oral part of the exam must be taken in all cases that either the instructor or the student requests it be taken (if a student wants to improve the grade of the written part of the exam). For example, a student who got 24 in the written test might want to skip the oral part, but also take it to improve the grade.
- The oral part of the exam, when mandated, needs to be taken in the same round (*appello*) of the written part of the exam. In all cases, the exam ends in the same round of a written part.

During the course of the lectures students will be allowed to practice solving problems on the computerized system from their homes and, in case they work on all the problems during the required time frame, they can gain up to 2 extra points. The bonus points will be added to the grade of the written exam, allowing them to have an improved grade for the written part of the exam.

Textbooks and Reading Materials

1. Schlesinger E., Algebra Lineare e Geometria, Zanichelli, 2017 (second edition)
2. Fioresi R.,Morigi M., Introduzione all'Algebra Lineare, Casa Editrice Ambrosiana, 2021 (second edition)
3. Notes of Lecures available on this platform.

Semester

Second semester (March-April)

Teaching language

Italian, English