UNIVERSITÀ DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Functional Analysis

2122-1-F4001Q075

Aims

Consistent with the educational objectives of the Master's Degree in Mathematics, the course aims to provide students with the knowledge concerning the definitions and the basic statements of the Functional Analysis. The skills needed to understand and analyze the main techniques and demonstration methods related to the theory, and the skills to apply them to face problems to different areas of Mathematics will be also provided. Particular emphasis will be placed on topological aspects.

Contents

Locally compact Hausdorff spaces. Spaces of continuous functions. Spaces L^p. Compactness in L^p and in C^0. Weak and weak* (weak star) topology. Compactness in the weak topologies. Riesz representation theorems.

Detailed program

Metric spaces, normed vector spaces, compactness of the closed ball and dimension.

Spaces of continuous functions. Urysohn Lemma and cut-offs. The Stone-Weierstrass theorem: density and separability. Compactness in the spaces of continuous functions: the Ascoli-Arzelà Theorem.

L^p spaces and their basic properties. Density and separability: the Lusin Theorem. Compactness in the L^p spaces: the Kolmogorov-Riesz Theorem. The canonical embedding of Frechet-Kuratowski for separable metric spaces.

Linear functionals and weak topology on a normed space. Sub-additive positively homogenous functionals. The

Hahn-Banach theorem: general form. Convexity and hyperplane separation. Mazur Theorem: weak and strong closure of convex sets.

The weak* (weak star) topology. Dual and bi-dual. The Banach-Alaoglu Theorem: weak* compactness of the closed ball in the dual space.

Reflexive spaces. Reflexivity in the L^p spaces. Uniform convexity and reflexivity. Weak and strong convergence in uniformly convex spaces. Kakutani and Eberlein-Smulyan theorems: weak compactness of the closed ball and reflexivity. Sequential compactness in the weak* topology.

Locally convex topological vector spaces. Convex hull and extremal points: the Krein-Milman theorem.

Real (and complex) valued measures. The Radon-Nikodym Theorem. Duality in the spaces of continuous functions: the Riesz Representation Theorem.

Prerequisites

Elements of the theory of abstract integration, elements of L^p space theory , elements of general topology. Basic knowledge of Banach spaces and Hilbert spaces.

Teaching form

Frontal lectures devoted to introduce the main theoretical concepts, to present detailed proofs of the theorems and to analyse explicit examples. Take-home exercises could be assigned in order to apply the theoretical notions in concrete situations.

Textbook and teaching resource

Bibliographic references

H. Brezis. *Functional analysis, Sobolev spaces and partial differential equations*. Universitext. Springer, New York, 2011
G.B. Folland. Real analysis. *Modern techniques and their applications.* A Wiley-Interscience Publication. John Wiley & Sons, Inc., New York, 1999.
W. Rudin. *Real and complex analysis*. McGraw-Hill Book Co., New York, third edition, 1987
T. B•uhler and D. A. Salamon. *Functional analysis.* volume 191 of Graduate Studies in Mathematics. AMS, Providence, RI, 2018.
Further material will be shared on the E-Learning site of the teaching.

Semester

First semester.

Assessment method

The exam is solely oral and consists of a colloquium with assessment. It is divided into a series of questions designed to verify the student's knowledge and mastery of the theorems with related demonstrations carried out during the course.

In the oral exam it is assessed whether the student has acquired the necessary skills to present a selection of the demonstrations carried out in the classroom, and, above all, the critical and operational knowledge of the definitions and results of the course, by illustrating examples and counter-examples.

Office hours

By appointment