



UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Analisi Geometrica

2122-1-F4001Q113

Aims

The aim of the course is to provide an introduction to the modern theory of the analysis on metric spaces, highlighting its geometric aspects and the bases of the differential calculus.

The expected learning outcomes include:

- the knowledge and the understanding of the fundamental definitions and statements, as well as of the arguments in some proofs; the knowledge and the understanding of some classes of fundamental examples to which the theory applies.
- the ability to recognize and analyze the (length, intrinsic or measure) metric spaces which can arise in different fields of pure and applied mathematics; the ability to determine the more relevant geometric and analytical features of a metric space and to develop a first order differential calculus on these spaces; the ability to clearly present the contents of the course, to manipulate some examples and to identify connections between the different topics covered in the course.

Contents

Basic notions and geometric aspects (curvature) of metric spaces.

Elements of first order differential calculus on measure metric spaces.

Detailed program

Part I. Metric spaces and curvature.

- Metric spaces: definition, examples, topology; Hausdorff measure and dimension.
- Length spaces, intrinsic metrics, geodesics, length and velocity; constructions and examples.
- Spaces of bounded curvature: some equivalent definitions of curvature bounds (from above or from below); angles and distance function; local and global curvature bounds.
- Convergence of metric spaces: uniform and Gromov-Hausdorff convergence.
- Some properties of metric spaces with positive curvature: volume growth, Hausdorff dimension; examples (cones, convex sets, ...); an overview of some compactness results.

Part II. Differential calculus on metric spaces of measure

- Measure metric spaces; doubling measures; covering lemmas: Vitali's and Lebesgue's theorem.
- Hardy-Littlewood maximal function: boundedness results.
- A review of Sobolev spaces in \mathbb{R}^n ; some equivalent definitions; Sobolev embeddings; Poincaré inequalities.
- Lipschitz functions: extension and density theorems; upper gradient; modulus of a curve family; capacity.
- Sobolev spaces on metric spaces: definition based on the maximal function; definition based on the upper gradient. Poincaré inequalities on metric spaces.
- An introduction to differential equations on metric spaces: energy minimization problems.

Prerequisites

Calculus in several variables, elements of measure theory, of Hilbert spaces and of L_p spaces.

A basic knowledge of the Sobolev spaces in \mathbb{R}^n can help to familiarize with some of the topics, but they are not necessary.

Teaching form

Lectures with blackboard.

Textbook and teaching resource

The main textbook are:

- D. Burago, Y. Burago, and S. Ivanov. *A course in metric geometry*, volume 33 of Graduate Studies in Mathematics. American Mathematical Society, Providence, RI, 2001 (for the 1st part of the course)

- J. Heinonen. *Lectures on analysis on metric spaces*. Universitext. Springer-Verlag, New York, 2001 (for the 2nd part of the course)

Supplementary textbooks and resources may be suggested during the course.

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Semester

II semester.

Assessment method

Oral exam. Mark out of thirty.

Part of the exam may consist of exposing an additional theme to those covered in the course, agreed in advance with the teacher. The remaining part aims at verifying the knowledge and the understanding of the topics (both theory and examples) presented during the course.

Office hours

By appointment.
