SYLLABUS DEL CORSO

Metodi Matematici per la Fisica Moderna

2122-1-F4001Q087

Aims

According to the educational objectives of the Course, the taught material aims to provide students with the *basic notions* regarding the definitions and the fundamental results for a geometric and topological approach to the study of classical field theory, with particular emphasis on classical vortex dynamics, ideal magnetohydrodynamics and quantum hydrodynamics. The course aims to provide also the necessary *competences* to understand and use standard techniques and the demonstration methods involved in the theory, as well as the *capabilities* to use them to solve exercises and tackle problems.

The expected outcomes include:

- the knowledge and understanding of the fundamental definitions and statements, as well as of the basic strategies of proof in classical geometric and topological field theory; the knowledge and understanding of some key examples where theory is fully applied;
- the ability to recognize the role that concepts and techniques from a geometric and topological approach play in various areas of mathematics (such as vortex theory, ideal magnetohydrodynamics, quantum fluids

Contents

Part I. Fundamentals of Green's potential theory and Kleinert's theory, fluid flows and diffeomorphisms, conservation theorems, Euler's equations, Helmholtz's conservation laws, Navier-Stokes equations, ideal magnetohydrodynamics, magnetic helicity.

Part II. Elements of knot theory, linking number and self-linking, topological interpretation of magnetic helicity, magnetic relaxation, Gross-Pitaevskii equation, topological defects in condensates, change of topology by

reconnection processes.

Detailed program

The course is divided into two parts, the first being of introductory and general character, the second focused on more specific topics of current research.

Part I. Fundamentals of potential thoery in terms of Green's identities, Kelvin's correction for multiply connected domains, fluid flows and diffeoporphisms, kinematic transport theorem, conservation theorems, decomposition of fluid motion, Euler's equations, vorticity transport equation, Helmholtz's conservation laws, Biot-Savart law, Navier-Stokes equations, energy dissipation, Burgers' steady vortex solution, Maxwell's equations, ideal magnetohydrodynamics, magnetic helicity, perfect and non-perfect analogous Euler's flows.

Part II. Elements of knot theory, fluid dynamic interpretation of Reidemeister moves, inflexional configuration and twist energy, linking number and self-linking, derivation of linking number from magnetic helicity, writhing and total torsion helicity, magnetic energy relaxation, groundstate energy spectra, ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

Prerequisites

Elements of differential geometry of curves and surfaces in three-dimensional space, elements of mechanics of continuum systems, balance laws in physics.

Teaching form

Standard blackboard lectures taught in English supported by lecture notes (in English) made available by the lecturer.

Textbook and teaching resource

Lecture notes (in English) made available by the lecturer during the course.

Semester

II semester.

Assessment method

Written exam paper (in Italian or English) based on 4 questions taken from a list of questions made available to students at the end of the course. Specific solutions must reproduce the material presented during the course, including detailed proofs of theorems and statements, complete with explicit computations. The final mark is expressed by 30 marks unit.

The written examination paper must show operational *capability* to tackle and solve the proposed questions by

using the acquired *knowledge* and the necessary *competence* to reproduce the topics presented during the course.

Office hours

Upon appointment, to be arranged with the lecturer by email contact: ⎯