UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Geometria e Fisica

2122-1-F4001Q079

Aims

The course has the aim of presenting the mathematical tools and the conceptual ideas that are required to understand the formulation of Einstein's gravitational field. In the second part of the course the same mathematical tools will be used to discuss some geometric aspects of the modern theory of integrable systems.

The expected learning outcomes include:

- the mastering of tensor calculus.
- the knowledge of the basic concepts of the general relativity.
- the knowledge of the basic concepts of the theory of integrable systems of hydrodynamic type.

Contents

Recalls of special relativity.

Tensorial calculus. Metrics, connections and curvature.

Elements of general relativity: the equivalence principle. Curved space-time. Einstein's equations.

Systems of hydrodynamic type. Differential geometric Poisson brackets. Pencils of flat metrics and bihamiltonian structures.

Detailed program

- Riemannian and pseudo-riemannian metrics. Minkowski space-time. Lorentz transformations.
- Recalls of the theory of surfaces. The first and the second fundamental forms. Gauss's egregium theorem. Gauss Peterson Mainardi Codazzi equations. Riemannian and pseudo-riemannian manifolds.
- Tensor fields : algebric theory. Algebraic operations on tensors. Lie derivative. Tensors in riemannian and pseudo-riemannian manifolds. Raising and lowering indices. Covariant derivative. Levi-Civita connection. Parallel transport and curvature. Geodesics and geodesic deviation.
- Elements of general relativity: the equivalence principle. Curved space-time. Einstein's equations.
- Systems of hydrodynamic type. Riemann invariants. Integrability conditions and generalized hodograph method.
- Differential geometric Poisson bracket. Bihamiltonian structures and flat pencils of metrics. An important exmple: the orbit space of a Coxeter group and the polynomial solutions of WDVV equations.

Prerequisites

The basic notions of Mathematical Analysis I and II, Linear algebra and Geometry, Physics I and II and Dynamical Systems and Classical Mechanics of Batchelor Degree are needed. The prior knowledge of the contents of the courses Mathematical Physics (for the second part of the course) and Geometry III (for the first part of the course) might be useful but it is not required.

Teaching form

Live lectures at the blackboard.

Textbook and teaching resource

Selected chapters from:

- B.A. Dubrovin, A.T. Fomenko, S.P. Novikov, "Modern Geometry - Methods and Applications. Part I. The Geometry of Surfaces, Transformation Groups, and Fields", Springer Graduate Texts in Mathematics
- R. d'Inverno, "Introducing Einstein's Relativity", Oxford University Press.
- N.M.J. Woodhouse, "General Relativity", Springer Undergraduate Mathematics Series.

Semester

First semester

Assessment method

Oral examination. During the oral exam the students will be asked to discuss some topics of the program and to illustrate their meaning with significant examples. The oral exam will evaluate the knowledge of the theoretical aspects of the course, as well as the ability to expose it in a well-organized and consistent way. One of the topics of the last part of the course (general relativity or integrable systems) can be replaced by a written report and a seminar on a subject in agreement with the lecturer.

Office hours

By appointment.