UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Geometria Differenziale

2122-1-F4001Q071

Aims

The aim of the course is to introduce the foundations of the theory of Riemannian manifolds, that is, manifolds endowed with a Riemannian metric, which consists in the assignment to each tangent space of a smoothly varying Euclidean product. The students will familiarize with the most basic concepts and techniques of differential geometry, moving from the foundational concept of Levi-Civita connection. Starting from the latter, the basic local curvature invariants and the notion of geodesic will be introduced. A key aspect which we propose to illustrate is the interplay between local aspects of the Riemannian metric, and the global topological structure of the underlying manifold.

The expected learning outcomes include the following:

- the knowledge and understanding of the basic definitions and statements, as well as of the basic strategies of proof in the theory in differential geometry; the knowledge and understanding of some of the key foundational examples of the theory;
- the ability to apply the acquired abstract knowledge to the solution of simple computational exercises and theoretical problems, referring in a precise and well-organized manner to the pertinent results being used; the ability to apply the theoretical background to the construction and discussion of simple examples and solution of exercises; the ability to expose and communicate effectively and clearly the theoretical content of the course.

Contents

Differentiable manifolds and Riemannian metrics, connections, curvature invariants, hypersurfaces and Lie groups, Riemannian submersions, Riemannian submanifolds, parallel transport and geodesics. Some notable global results, such as, time permitting, the Theorems of Gauss, Hopf-Rinow, Hadamard, Bonnet-Myers.

Detailed program

Brief recalls on differentiable manifolds; Riemannian metrics; the fundamental theorem of Riemannian Geometry and the Levi-Civita connection; the curvature tensor and local curvature invariants; examples: Lie groups, hypersurfaces, metrics with rotational symmetry; shape operator; equations of Gauss and Codazzi-Mainardi; Theorema Egregium; Riemannian submersions and the formula of Gray and ‘O Neill; the Hopf map; parallel transport and geodesics, existence and uniqueness; examples; the exponential map; normal coordinates,; Jacobi vector fields; some global results, such as the Theorems of Gauss, Hopf-Rinow, Hadamard, Bonnet-Myers.

Prerequisites

The content of the courses of the first two years of the Laurea Triennale in Mathematics should in principle be an adequate background. However, those students who do not have the basic notions on differentiable manifolds offered in, say, the course of Geometry III of the same program should expect to do some parallel extra work, since the discussion on prerequisites will be limited to some brief recalls.

Teaching form

Lectures: 8 CFU

Textbook and teaching resource

Main references:

M. Do Carmo, Riemannian Geometry, Birkhauser

Recoomended reading:

M. Do Carmo, Differential forms and applications, Springer Verlag 1996;

P. Petersen, Riemannian Geometry, Springer Verlag 2006

Semester

1st semester

Assessment method

During the course, two written partial tests will be offered, each referred to one half of the course. Each partial test will consist of a balanced flexible combination of computational exercises and theoretical questions. The exercises and theoretical questions in these tests will be along the lines of those offered in the practical and theoretical tests of the regular exam sessions (see below). The two partial tests will contribute equally to the final grade. To pass the exam through the partial tests, the student needs to pass each of them, thus obtaining a grade of at least 18/30 in both.

Alternatively, students may pass the exam through the regular exam sessions that follow the end of the course, and exactly the same pattern will be offered in every exam session. Thus, each session comprises two written tests, each referred to one half of the course, and consisting of a balanced combination of computational exercises and theoretical questions. The theoretical questions will involve definitions, statements of theorems, proofs, construction of examples and counterexamples, and simple theoretical problems.

The exercises will measure the student's ability to master the acquired formalism and apply it to some simple computations, to build on the acquired theoretical knowledge, and to invoke it in a pertinent and precise manner.

The theoretical questions will evaluate the knowledge and understanding of the conceptual framework of the course, as well as the ability to expose it in a well-organized, consistent and effective manner.

In order to pass the exam in one of the regular sessions, the student needs to obtain a grade of at least 18/30 in each of the two tests, which will contribute equally to the final grade. The two tests needn’t be undertaken in the same session. It is also allowed to pass one the tests during the course and the other in a regular exam session.

To each exercise/theoretical question (or problem) a maximum partial grade will be assigned by the commission, depending on its difficulty and length. In the evaluation, every student will be given a grade in correspondence to each exercise/theoretical question (or problem) up to the maximum one, measuring the exactness, the completeness, the rigour, the clarity and the overall coherence of the development.

The exact subdivision of the course in two parts will be communicated well in advance during its duration.

Office hours

Upon appointment