UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Metodi e Modelli Stocastici

2122-1-F4001Q106

Aims

To provide a selection among methods, concepts and advanced models of probability theory and stochastic processes, from a theoretical and practical point of view.

At the end of the course, students will have acquired the following:

- *knowledge*: a selection among advanced results of probability theory (large deviations), stochastic processes (continuous-time Markov chains) and stochastic modeling (random graphs);

- *competence*: operational understanding of the probability language and advanced proof techniques (e.g. coupling);
- *skills*: ability to apply theoretical notions to the solution of exercises and the analysis of problems and models.

Contents

The course starts with an introduction to the **Poisson process** which is the most important example of continuous-time stochastic process having discrete states and the starting point to study more general **continuous-time Markov chains**. In the second part of the course we present some results in **large deviation theory** providing tools to investigate the probability of rare events at exponenial scale. In the third part of the course we shall study topics related to **random walks**, a fundamental and rich object in probability. In the last part of the course we will discuss the theory of **random graphs**, a research topic that is receiving great attention.

Detailed program

1. Poisson process

- Introduction to point processes
- Poisson process
- Asymptotic properties

2. Continuous-time Markov chains

- Semigroups and generators on countable spaces
- Continous-time Markov chains
- Strong Markov property
- Convergence to equilibrium

3. Large deviations

- Cramer's Theorem

- Relative entropy and Sanov's Theorem
- Large deviations principle
- Application: Curie-Weiss model

4. Random walks

a) Random walks

  - Simple random walk on the integers
  - Polya's Theorem for simple random walks on the square lattice
  - Kesten-Spitzer-Whitman's Theorem on the range

b) Random walks in random environments (RWRE)

  - Dirichlet problem for random walks on graphs
  - Solomon's theorem for RWRE on the integers

*c) Countable Markov chains

  - Lyapunov function criteria for recurrence and transience

- Applications
    - Another proof of Polya's Theorem
    - Branching processes with migration
    - Excited random walks

5. Random graphs

- Introduction to random graphs
- Erdos-Renyi model
- Connectivity and giant component in the Erdos-Renyi model

**we may not cover some (or all the) material in this topic, depending on the speed of classes*

Prerequisites

The knowledge, competences and skills taught in classical probability and stochastic processes courses (random variables, martingales, conditional law) as well as those taught in mathematical analysis courses.

Teaching form

Lectures and recitations in the classroom, divided into:

- theoretical lectures, focused on the knowledge of definitions, results and relevant examples;
- practical lectures, focused on the skills necessary to apply the theoretical knowledge and competences to both the analysis of models and the solution of exercises.

Textbook and teaching resource

Reference textbooks:

- E. Pardoux, *Markov processes and applications*, Wiley Series in Probability and Statistics (2008)
- F. den Hollander, *Large Deviations*, Americal Mathematical Society (2008)
- R. van der Hofstad, *Random Graphs and Complex Networks*, Volume I, Cambridge University Press (2017)
- S. Asmussen, *Applied Probability and Queues*, Springer (2003)
- Lecture notes of the course "Topics in Random Walks" by Tal Orenshtein in 2019 at TU Berlin

- Q. Berger, F. Caravenna, P. Dai Pra, Probabilità: un primo corso attraverso esempi, modelli e applicazioni (II edizione), Springer (2021)

Other material:

- Lecture notes
- Other references / notes by the teacher

Semester

Spring term

Assessment method

The exam consists of two parts*:* **individual assignment of exercises** contribuiting one sixth to the final grade, and an **oral exam** contribuiting five sixths to the final grade, which will be converted as a 30 point score.

The **individual assignment of exercises** consists in the resolution of some exercises proposed during the course, which have to be solved autonomously by the students and due (at least) one week before the oral exam. This examination tests the continuity of learning as well as practical skills.

The **oral exam** consists in an interview lasting about 30-60 minutes and tests the knowledge of definitions, statements and examples presented during the course, as well as presentation skills related to a selection of topics and detailed proofs.

There will be 5 exam sessions (two between June and July, one in September and two in February).

Office hours

By appointment