UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Stochastic Processes

2122-1-F4001Q059

Aims

In line with the educational objectives of the Master Degree in Mathematics, the course aims at providing the *knowledge* about the fundamental concepts and statements of the theory of stochastic processes in discrete time. It will also build the *skills* needed to understand and use the most important proving arguments and techniques in the theory and the *ability* to solve exercises and deal with problems exploiting them. Particular emphasis will be put on the theory of martingales.

Contents

Conditional law and conditional expectation. Martingales in discrete time. Introduction to the asymptotic behavior of Markov chains. Financial markets and Martingales. Examples and applications.

Detailed program

- Conditional law and expectation. Definitions and properties. Existence of conditional expectation of a random variable with respect to a sigma algebra. Fundamental properties: tower property, Jensen inequality, freezing. Limit theorems.
- Discrete-time Martingales. Definition and examples (sums of independent centered r.v.s, products of independent r.v.s with expectation 1, closed martingales). Integral of a predictable process. Stopped Martingales. Optional stopping theorem. Applications: first hitting time of a random walk on Z; the gambler's ruin problem. Upcrossing Lemma. Almost certain convergence of martingales bounded in L^1 norm. Martingales bounded in L^2 norm. Uniform integrability and convergence in L^p. Maximal inequality. Doob's inequality. Examples: Galton-Watson branching processes. Absence of arbitrage in discrete time binomial markets.

- Asymptotic behavior of Markov chains. Definition and properties. Links with martingale and harmonic functions. Invariant measures: existence and uniqueness in the irreducible and positively recurrent case. Ergodic theorem and law of large numbers.
- Financial markets with discrete time. Arbitage and equivalent martingale measure.

Prerequisites

Knowledge of differential and integral calculus for functions of one and more real variables, as well as measure-theoretical probability theory is needed. It is also useful to know definitions and basic properties of L^p spaces and Hilbert spaces.

Teaching form

Lectures in the classroom, divided into: theoretical lessons in which the knowledge about definitions, results and relevant examples is given and other lessons in which we try to give the skills and abilities needed to use the previous notions to solve exercises and to deal with problems (also related to extra-mathematical applications).

Textbook and teaching resource

- D. Williams, Probability with Martingales, Cambridge University Press (1991).
- E. Pardoux, Markov Processes and Applications, Wiley Series in Probability and Statistics (2008).
- Lecture notes (available on the e-learning platform)
- Written tests from previous years, with detailed solutions (available on the e-learning platform).
- List of proofs that may be requested during the oral examination (available on the e-learning platform).

Semester

First (fall) semester.

Assessment method

Written and oral exam. Mark out of thirty.

The written test evaluates the operational *ability* to solve exercises, it receives a mark out of thirty. It is necessary to obtain an evaluation of at least 16/30 in the written test to access the oral exam, which evaluates the *capacity* to present a selection of proofs and, above all, the critical and operational *knowledge* of the definitions and results presented during the course, also by means of examples and counterexamples. The final evaluation will result from the average between the evaluation of the written test and that of the oral examination. The exam is passed if the evaluation is at least 18/30.

There will be 5 exam sessions (January, February, June, July, September).

Office hours

By appointment.