UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Topologia Differenziale

2122-1-F4001Q111

Aims

The scope of this course is the continuation of the study of Geometry along the path started in the Laurea Triennale (Bachelor). While it is not a strict prerequisite to the other courses in Geometry, which can be taken independently, it aims to unify and connect the different themes and perspectives developed in them.

Differential topology studies the interplay between the differential structure and the topological properties of smooth manifolds. Differential forms link these two aspects, as they are at the same time an offspring of the smooth structure and the constituent of de Rham cohomology. Differential topology is thus also a natural starting base to explore more abstract aspects of algebraic topology. These techniques also yield a geometric approach to intersection theory.

The expected learning outcomes include the following:

- the knowledge and understanding of the basic definitions and statements, as well as of the basic strategies of proof in the theory of differential topology; the knowledge and understanding of some of the most relevant basic applications and examples of the theory;
- the ability to apply the acquired abstract knowledge to the construction and discussion of simple examples and solution of exercises; the ability to expose and communicate effectively and clearly the theoretical content of the course.

Contents

De Rham Theory for smooth manifolds; transversality and intersection theory.

Detailed program

Part I:

- De Rham cohomology. Mayer-Vietoris sequence. Poincarè Lemmas and the degree of a proper map. Poincarè duality on orientable manifolds. Kunneth's Formula and Laray-Hirsch's Theorems. Vector bundles and Thom isomorphism. The Euler class and Euler characteristic.

Part II:

- Transversal maps, intersection of transversal varieties. Applications: classifications of 1-manifolds and Brower fixed point Theorem. Transversality Theorems. Intersection numbers mod 2 and degree of a map mod 2. Intersection Theory for oriented varieties.

Prerequisites

The content of the courses of Analysis I, Linear Algebra and Geometry, Geometry I. The basics on differential varieties and differential forms (as content of Geometry II and Geometry III). Brief recalls will be offered as needed.

Teaching form

Front lessons at the blackboard.

Textbook and teaching resource

Reference texts:

R. Bott e L. Tu, Differential Forms in Algebraic Topology, Springer-Verlag

V. Guillemin, P. Haine, Differential forms, World Scientific Publishing Co.

V. Guillemin e A. Pollack, Differential Topology, Prentice Hall

Semester

Second semester

Assessment method

The exam comprises two written tests, followed by an oral discussion. Each test is referred to a single part of the course (I and II) and will evaluate the knowledge and understanding of the conceptual framework of the course, as well as the ability to expose it in a well-organized, consistent and effective manner. In each test there will be questions involving definitions, statement's of theorems, proofs, construction of examples and counterexamples, and simple theoretical problems. Each test will be assessed independently and will contribute equally to the contribution of the final grade; in order to successfully complete the exam the students need to obtained a grade of at least 18/30 in each test.

The two tests can be taken in different sessions. It is possible to enroll in both written tests, but only the second one gives the registration of the vote. The date of the oral discussion will be announced after the correction.

Office hours

Upon appointment.