UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Derivatives

2122-1-F1601M051-F1601M056M

Learning objectives

1) Knowing the simplest types of derivatives instruments and understanding their meaning and financial uses

2) Understanding the concept of model of a financial market and its use in valuing derivative instruments

3) Knowning in full details the various models studied and the derivations of the various pricing formulas

4) Being able to apply the various models to the pricing of a generic contingent claim.

Contents

- Preliminaries on options

- Multiperiod Binomial model

- General one-period model

- Continuous time models

- Black Scholes model

- Beyond Black-Scholes

- Monte carlo method

- Vasicek model

- Merton model

Detailed program

Introduction to derivatives

Forward, futures, options. Payoffs and replication of the forward contract. Forward price and spot-forward parity. Put-call parity. Payoffs of combinations (spread, butterfly, strangle, straddle),

Convexity of the call price as a function of the strike. Superreplication and subreplication. Merton bounds. American options and value of early exercise. Qualitative discussion of the factors affecting option prices.

Binomial models

Pricing a generic payoff in the one-period binomial model. Two-period binomial models and american options. Multiperiod binomial models: valuation of a generic payoff and formula for a call option. Choice of the parameters u and d and historical volatility matching.

Black Scholes formula as a limit of the binomial model.

One period models

One period models. Payoff matrix. Replicability, completeness, sufficient condition for completeness. Definition of arbitrage opportunity. State price vectors and first fundamental theorem of valuation. Second fundamental theorem of valuation. Superreplication and subreplication as linear programming problems. Dual formulation.

Continuous-time models

Brownian motion: definition and first properties. Ito processes: definition and examples. Ito formula: drift and volatility of a transformed process. Geometric Brownian motion, lognormal distribution.

Black-Scholes model

Derivation of the Black-Scholes differential equation. Elementary solutions, linearity and superposition principle. Derivation of the Black-Scholes formula as the risk neutral discounted expected value of the payoff. First properties of the BS formula; sensitivities with respect to the parameters and computation of the greeks. First extensions of

the BS model; dividends. Approximation of the BS formula for ATM short maturity options. Empirical fit of the BS formula. Implied volatility and volatility smile.

Beyond Black-Scholes
Implied volatility and smiles. Basic notions on local volatility models and on stochastic volatility models.
The VIX Index.

Montecarlo method
Random number generation. Simulating stochastic processes. Pricing a derivative and computing his greeks with the Montecarlo method.

Vasicek model
Short rate dynamics. Stochastic integral of a deterministic function with respect to the Brownian motion. Derivation of the distribution of the short rate. Derivation of the yield curve and example of calibration.

Merton model
Credit risk. Merton model: equity as a call option on the asset value. Determination of the risk neutral probability of default. Determination of the spread curve for a defaultable bond.

Prerequisites

The notions required for the access to ECOFIN.

Teaching methods

All lectures will be delivered in presence and at the same time broadcasted.

Assessment methods

Written exam with oral integration that can be required either by the student or by the professor.

Textbooks and Reading Materials

- Lectures notes and slides
- J. Hull "Opzioni e futures"
- J. Cox, M. Rubinstein "Option markets"

Semester

First Semester

Teaching language

Italian