UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Mathematics for Physics

2223-2-E3001Q075

Aims

Generalize the basic notions of analysis to the complex plane. Introduce the mathematical concepts required in the formulation of Quantum Mechanics.

Contents

1) Complex analysis. Holomorphic functions. Power series in the complex domain. Chauchy theorem. Laurent series. Residue theorem. Analytic continuation.

2) Review of linear, topological and Banach spaces. Hilbert spaces. Lp spaces. Orthonormal basis. Fourier series. Linear operators in Hilbert spaces and their properties. Self-adjoint and unitary operators. Spectral decomposition. Fourier transform. Laplace transform.

3) Distributions.

Detailed program

The course will cover the following topics, not necessarily in the given order, with applications to problems and differential equations of interest in physics**:

Complex analysis: The complex plane. Complex functions of complex variable. The derivative of a function in C. Cauchy-Riemann conditions. Integration on the complex plane. Cauchy theorem. The behaviour of a complex function close to an isolated singularity. Laurent series expansion. Residue theorem. Computational techniques for integrals along the real axis by using the analytic continuation in C. Analytic continuation and multivalued functions.

Funtional spaces: Summary of the main properties of topological spaces, metric spaces, Banach spaces. Hilbert spaces. Orthonormal basis. Fisher-Riesz theorem. Lp spaces. Important examples of orthonormal basis: Fourier series, Hermite, Legendre, Laguerre polynomials. Linear operators in Hilbert spaces and their properties. Continuous and bounded operators. The norm of an operator. Spectral problem, classification of the eigenvalues. Definition of eigenfunctions. Self-adjoint and unitary operators. Eigenvalues and eigenvectors of self-adjoint operators. Theorem of spectral decomposition. Fourier transform in L1 and L2 its properties. Lapace Transform

Distributions: brief introduction to the theory of distributions. Examples of distributions. Operations on distributions.

Prerequisites

Contents of Analysis I, II and "Algebra and Geometry".

Teaching form

Class lectures (5 CFU) and tutorials (3 CFU).

Textbook and teaching resource

Main references:

Michela Petrini, Gianfranco Pradisi, Alberto Zaffaroni, A Guide to Mathematical Methods for Physicists With Problems and Solutions
World Scientific

J. Bak, D.J. Newman, Complex Analysis, Springer

L. Debnath, P. Mikusinski, Hilbert spaces with applications, Elsevier

More advanced references and topics:

Michela Petrini, Gianfranco Pradisi, Alberto Zaffaroni, A Guide to Mathematical Methods for Physicists Advanced Topics and Applications
World Scientific

Walter Rudin, Real and Complex Analysis, Mc Graw Hill (avanzato)
Esercises

M.R. Spiegel, Complex variables, Schaum Outline Series

M.R. Spiegel, Fourier Analysis, Schaum Outline Series

Other solved exercises and previous exams will be available on the e-learning page

Semester

Second semester

Assessment method

The exam consists of a written (exercises) and an compulsory oral part.
The oral part concerns the entire program, including exercises and applications, but also the discussion of the written exam.
The exam has to be completed during the breaks and in the same session (summer=Jun-Jul-Sep or winter=Jan-Feb) of the written exam .

During the course, two partial written exams are proposed (containing exercises, problems and open-ended theory questions) which will be graded in 30 points units.
The first partial exam will focus on Complex Analysis, while the second will cover Functional Spaces and Distributions.
Passing the partial exams (>=15/30) is equivalent to passing the written exam in the summer session.
If the average of the two grades is greater than or equal to 21/30 the student is exonerated from the oral exam, unless the oral exam is explicitly requested by the student or by the teacher.

In case the oral examination takes place the final score will be the average of the written and oral part.
Therefore, the final mark could be lower than the one of the written exam.

Office hours

On appointment by e-mail

Sustainable Development Goals