UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Mathematical Analysis III

2223-3-E3501Q056

Aims

The course aims at providing the knowledge about the fundamental concepts and statements of advanced mathematical analysis. It will also build the skills needed to understand and use the most important proving arguments and techniques in the theory and the ability to solve exercises and deal with problems exploiting them.

Contents

Banach Spaces. Lp spaces. Hilbert spaces. Fourier series. Convolution. Fourier transform. Baire's Theorem. Open mapping Theorem. Banach Steinhaus Theorem. Dual space. weak convergence.

Detailed program

Definition of Banach space. Examples.
Definition of L ^ p (X, ?), ? positive measure.
Holder and Minkowski inequalities.
Completeness of L ^ p (X, ?).
Inclusions of spaces L ^ p (X, ?), finite ?.
Inclusions of spaces L ^ p (Z).
Relations between pointwise convergence, convergence in Lp, and in measure.
Density of Cc (Rn), Coo (Rn) and of the Schwartz space in L p (Rn).
Duality of Lp spaces (only statement).
Hilbert spaces.
Inner product.
Cauchy-Schwarz Inequality.

Hilbert space.
Points of minimum distance from a closed convex.
Projection theorem.
Bessel inequality.
Complete orthonormal systems.
Parseval formula.
Gramschmidt process.
Fourier series for functions on the thorus
Dirichlet kernel.
Convergence in L2.
Pointwise convergence.
Linear operators between normed vector spaces.
Dual space.
Baire's theorem.
The Banach-Steinhaus Theorem.
Divergence of the Fourier series.
Open Mapping Theorem.
Closed Graph Theorem.
Non surjectivity of the Fourier transform from L 1 (T) into c_0 (Z).
Weak convergence.
Fourier transform in Rn.

Fou

Prerequisites

Topology. Linear algebra. Differential calculus. Integral calculus. Measure theory. Complex numbers.

Teaching form

Lectures in the classroom, divided into: theoretical lessons in which the knowledge about definitions, results and relevant examples is given and other lessons in which students solve exercises at the blackboard showing their abilities to use the previous notions to deal with analitical problems.

Textbook and teaching resource

G.B. Folland "Real Analysis"
L. Grafakos "Classical Fourier Analysis"
W. Rudin "Real and Complex Analysis"
W. Rudin "Functional Analysis"
E.M. Stein R. Shakarchi "Functional Analysis"
E.M. Stein R. Shakarchi "Fourier Analysis"

Notes

Semester

Second semester

Assessment method

Written and oral exam.

Written exam

The written exam consists of exercises aimed at verifying the understanding of the course contents, the ability to apply the learning demonstration technique, the exposition clarity . Each exercise will be given a maximum partial score, due to its difficulty and length. In the evaluation of the student a score will be assigned based on the accuracy, completeness, rigor, clarity and organic nature of the performance. The maximum grade for the written exam is 33.
The proposed exercises are in line with those carried out during the lessons.

The student is admitted to the oral exam with an evaluation of at least 16.

The oral exam consists in a discussion of the written exam and in theoretical questions (definitions and theorems with proofs). In the oral exam the knowledge and understanding of the course content will be evaluated, as well as the ability to organize a coherent and punctual exhibition in a lucid, effective and well-structured manner.

The final grade is given by the grade of the written exam to which points are added or subtracted in the oral exam.

Office hours

By appointment.

Sustainable Development Goals

QUALITY EDUCATION