UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Mathematical Statistics

2223-3-E3501Q062

Aims

Knowledge of the basic tools of inferential statistics.

Acquire the ability of applying this knowledge to situation where we have a sample of observations, in order to provide, after a proper choice of the random model, estimates of the unknown parameters and reasonable oinions on the properties of these parameters.

Contents

Statistical models. Inferential statistics: estimators, confidence intervals, hypothesis testing.

Detailed program

The program is the same for attending and non-attending students

Introduction to Statistics

1. Target population, random sample. Sampling problem. Statistics. Density dependent on unknown parameters.

Point estimation of parameters

1. Estimator, unbiased estimator, mean square error, consistency in quadratic mean, necessary and sufficient condition for the consistency in quadratic mean of a sequence of estimators.

2. Estimators for the moments of a VA: sample moments. Sample mean.
3. Unbiased estimator of the variance: sample variance.
4. Method of moments for the construction of estimators.
5. Likelihood function. Maximum likelihood estimator.
6. UMVUE, the lower limit of variance (Cramér-Rao).
7. Invariance property of maximum likelihood estimators. Asymptotic properties of maximum likelihood estimators.
8. Sampling from Normal random variables: the law of the sample mean. Square law of a Normal (0,1): chi-square law with 1 degree of freedom. Law of the sum of squares of independent standard normals: Chi-square law with k degrees of freedom. Law of the sample variance of a normal sample. Student's t law.

Confidence intervals

1. Definition, confidence level.
2. Intervals for the mean of the normal population (known or unknown variance).
3. Intervals for the variance of the normal population (mean known or unknown).
4. Pivotal quantity and its use for the calculation of confidence intervals.
5. Confidence intervals for large samples (especially for frequencies or parameters of Bernoulli).
6. Pivotal quantity for samples from the absolutely continuous laws.

Hypothesis testing

1. Tests for statistical hypothesis, non-randomized tests and the critical region. Significance level, the p-value. Uniformly more powerful tests.
2. Test for a mean of the normal population (variance known or unknown).
3. Test for the variance of a normal population (mean known or unknown).
4. Test for difference of means for normal populations.
5. Test on a frequency and on two frequencies (large sample).
6. Test of simple and generalized likelihood ratio. Neyman-Pearson Theorem.
7. Pearson chi-square test for adaptation (with or without parameters estimated).
8. Pearson chi-square test for Independence.

Linear regression

1. Simple and multiple linear regression: definition, interpretation, testing.

Prerequisites

Mathematical analysis I and II, in particular integral calculus.

Basic probability: laws of discrete and continuous random variables. Expected value and variance. Law of functions of random variables. Independence. Convergence of sequences of random variables.

Teaching form

Lessons and exercises will be provided in the classroom.

In any case students are advised to exercise at home with the exercises which are provided through the elearning platform (some of those exercises are discussed during the lessons and are a model for the exercises which are

given during examinations).

Textbook and teaching resource

Textbook:

Introduzione alla statistica di A.M.Mood, F.A.Graybill, D.C.Boes, 1991, McGraw-Hill Italia,ISBN: 9788838606618

Other material: slides of the lessons and the exercises on elearning.unimib.it

Semester

Second semester.

Assessment method

Written exam with

1. multiple choiche questions (on the theoretical aspects described in the course)
2. open questions (on the theoretical aspects described in the course, includeing the requirement of writing definitions and statement and proof of theorems)
3. written exercises: application of the theoretical concepts and of techniques like the ones used in the exercises which are assigned in the classroom and at home.

The composition of the written exam may vary in the proportion of the three parts in different sessions, but all the three parts will always be represented. The written exam usually last from a minimum of 1h30' to a maximum of 2h20' (depending on the length of the questions/exercises). In the multiple choice questions we evaluate the capacity of recognizing the correct answers among wrong answers, and the capacity of understanding under which circumstances some properties of the objects, studied in the course, are valid or not. In the rest of the written exam we evaluate the correctness of the answers, the clarity and their completeness. We also evaluate the capacity of discussiong when certain statistical inference methods are more appropriate and when they are only an approximation.

Some open questions on the theoretical aspects are inserted in the written exam and the oral examination may be required by the student and/or the teacher and is a discussion based on the written examination, on the subjects treated during the lessons and eventually on the subject of the linear regression. In the oral examination we evaluate the same qualities of the answers of the written part. Students with an insufficient evaluation but larger or equal to 16/30 may ask to be evaluated orally, and so may all the other students with positive evaluation in the written examination. In case of an oral examination, its mark will have a weight of 1/4 (and 3/4 for the written part). Usually sufficient marks (including the maximum of the gradings) may be confirmed without an oral examination, but the teacher can request an oral examination in all the situations where she judges necessary to ask for explanations on the written exam.

During the year there are 5 examination, usually in the following months: June, July, September, November and January (or February).

Office hours

By appointment.

Sustainable Development Goals