UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Algebra I

2223-1-E3501Q003

Aims

Introduce students to methods and basic contents of algebra. Fundamental structures like sets, groups and rings will be studied. Time permitting some rudiments of programming languages as Magma, Gap and Mathematica will be imparted.

At the end of lectures students will be requested to solve both routine exercises and more sophisticated problems explotiong the theoretical contents of the course.

Contents

Sets, Relations, Operations; Modular Arithmetic; Elements of Group and Ring Theory; Polynomials;

Detailed program

A) Sets, Relations, operations: Axiom of choice; order relations (Zorn's Lemma); equivalence relations; homomorphism theorems for sets; congruences.

B) Arithmetic properties of the set of integers Z; modular arithmetics; residue classes.

C) Basics of Group Theory; subgroups, subgroup generated by a subset; cyclic groups; cosets; Lagrange's Theorem; congrunces in a group; normal subgroups; group homomorphisms and quotient groups; main theorems on homomorphisms; automorphisms; direct and semidirect products; symmetric and alternating groups; permutation groups; group actions (G-sets); regular representation; Cayley's Theorem; conjugation action; orbits: examles and applications; Sylow's Theorems.

D) Basics on Ring Theory: domains, division rings, fields; ring homomorphisms: ideals, quotint rings, elementary theory of homomorphisms; Chinese remainder Theorem; divisibility in a domain; embeddings of domains into fields; prime and maximal ideals; Euclidean, principal ideal and unique factorization domains; Gaussian integers.

E) Polynomial algebras: polynomials in one variable over a field; decomposition into irreducible factors.

Prerequisites

Basic notions of high school algebra and analysis

Teaching form

The course is organized in Lectures (48 hours, 6 CFU) and Exercise classes (24hours, 2CFU). Definitions, results and relevant theorems will be presented in Lectures, providing examples and problems making use of the notions introduced. Exercises related to the subject matters covered in the lectures are presented and solved during Exercise classes. In order to encourage student participation, some exercises are left for the students to solve.

A tutor will aid the students in solving the tests published on WIMS website https://wims.matapp.unimib.it/

Textbook and teaching resource

Textbook: Latex written Notes and Tablet taken lectures are available on this platform.

Suggested readings:

- Aschbacher, Finite Group Theory 2?? ed, Cambridge University Press, Cambridge, 2000.
- Childs, A Concrete Introduction to Higher Algebra 3?? ed, Undergraduate Texts in Mathematics, Springer, New York, 2009.
- Jacobson, Basic Algebra I, Freeman & Co, 1985
- Machi, Gruppi, Springer-Verlag, 2007

Exercise Books:

- Alzati, Bianchi, Cariboni, Esercizi di matematica discreta, Pearson, 2012
- Chirivi', del Corso, Dvornichich, Esercizi scelti di algebra Vols. 1 e 2, Springer, 2017

Semester

Second semester

Assessment method

In order to access the written assessment one has to pass a computer assisted exam.

This requires inscription to the WIMS platform https://wims.matapp.unimib.it/

There 12 tests are available (one for each week of lectures). They will be gradually activated.

Their resolutions will allow you to tune in with the course contents. Moreover the first part of the exam will consist of a few

exercises selected among those of all tests.

At the end of the lectures a bonus of xx will be assigned to a yy score according to following table:

- xx=2 if 27<yy<=30;
- xx=1.5 if 22<yy<=27;
- xx=1 if 18<=yy<22.

The bonus will be valid until March.

The examination is divided in five phases:

1. Multiple choice test to ascertain that basic concepts have been assimilated. Exact answers are expected (max. 10 points)
2. If a satisfactory score is reached the student is allowed to give a written exam consisting in the solution of a few routine exercises (max. 10 points)
3. Assessment of an exercise where skills of exploiting theoretical contents in problem-solving will be evaluated (max. 6 points)
4. Request to describe one of the main Theorems of the course providing hints of proof and examples (max. 4 points)
5. The oral examination requires the exposition of statements and proofs of the theorems, the definitions, the examples / counterexamples and the calculation techniques.
   It will be mandatory if the previous score is

Office hours

By appointment via e-mail.

Sustainable Development Goals

QUALITY EDUCATION | GENDER EQUALITY