UNIVERSITÀ DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Matematica I

2223-1-E2702Q001

Aims

- To understand the ideas and techniques of differential and integral calculus for real functions of one real variable.
- To acquire the ability for critical and autonomous elaboration of the fundamental concepts.
- To acquire the computational skills on the base of problems and exercises solved both under the supervision of the teacher and independently.
- To acquire the ability for rigorous exposition of the theoretical knowledges and of the solutions to problems and exercises.

Contents

Real numbers, operations and their properties. Elementary functions, properties and their graphs. Numerical sequences, limits of sequences, forms of indecision. Comparison of infinities. Numerical series, convergence tests. Absolute convergence. Function limits. Continuity. The derivative. Theorems of differential calculus. The Taylor's theorem. Primitive functions and indefinite integral. The Riemann Integral. Generalized integrals.

Detailed program

Real numbers. Natural numbers N, and the ring of relative integers Z. Induction principle. The field Q of rational numbers. Properties and their inadequatecy: the equation $x^2 = 2$ has no solution in Q. The real numbers field R. Decimal representation. The real axes, ordering. Intervals. Neighbourhood. Absolute value. Bounded sets in R. Maximum and minimum. Supremum and infimum, completeness. Roots, powers and logarithms.

Real functions of a real variable. Definition. Domain and range. Graph of a function. Elementary functions:

powers, exponentials, logarithmic functions. The sequence as a function whose domain is the set N. Bounded function. Maximum, minimum, superior, inferior of a function. Properties of a real function: injectivity, increasing, decreasing, monotone, convex, concave, even, odd. Extremal points, absolute extremals and relative extremals. Understanding the properties of the given definitions by reading the graph. Composite function, inverse function. Periodic functions, trigonometric functions and their inverse. Solving the inequalities by inversion of injective and monotonic functions.

Complex numbers. The field C of complex numbers: algebraic form, operations, equality. Representation in the complex plane. Polar coordinates, modulo and argument, trigonometric form, exponential form. De Moivre's formula. Formula of the n-th roots of a complex number * (solutions in C of the equation z? = w). The fundamental theorem of algebra.

Limits. Limits of sequence, of functions. Properties: uniqueness of the limit, permanence of the sign, existence of the limit for monotonic functions. Comparison test. The ratio test. Operations with limits, indecision forms. The limit e. Notable limits. Symbol of asymptotic Landau. Order of an infinitesimal / infinite, compared to a sample.

Numerical series. Sequence of partial sums. Convergent, divergent, irregular series. Geometric series, Mengoli series, harmonic series. Necessary condition of convergence. Series with positive terms: their regularity and convergence tests: comparison, asymptotic comparison, root and ratio tests. Series with alternating signs and Leibniz test. Simple and absolute convergence.

Continuity. Continuous function at a point, on a set. Classification of discontinuities. Operations between continuous functions, continuity of composite function. Properties of continuous functions in a closed and bounded interval: Weierstrass theorem, existence of zeros, Darboux (or intermediate values). Continuity and monotony. Continuity of the inverse function.

Differential calculus. The derivative and its geometric interpretation. Equivalence between derivability and differentiability for functions of a variable. Equation of the tangent line. Points of non-derivability. Continuity and derivability. Calculation rules for derivatives. Stationary points. Theorems of the differential calculus: Fermat, Rolle, Lagrange and its corollaries, examples and counterexamples. The theorems of De l'Hòpital. Higher order derivatives. Polynomial approximation: Taylor formula, Peano remainder and Lagrange remainder. Convexity and inflection points. Use of the derivatives of higher order to establish the nature of a stationary point. Asymptotes. Study of the graph of a function. Primitive functions and indefinite integral. Elementary methods for the search for a primitive. Integration by parts, by substitution (change of variable). Integration of rational functions.

The Riemann integral. Definition and its properties. The integral average theorem. Integral function, integration and differentiation. The fundamental theorem of calculus. Generalized integrals, definitions and examples.

Prerequisites

Sets operations, union, intersection; membership and inclusion. Operations and comparison between real numbers, sorting. Properties of powers. Second-degree equations. Binomial expansion. Polynomials, division between polynomials, root of a polynomial, Ruffini's rule. Factoring. First and second degree inequations, rational inequalities. Cartesian coordinates. The line, the parabola, the circle. Degrees and radians. Elements of trigonometry. Systems of first degree equations.

Teaching form

Lectures of theoretical slant and exercises sessions.

Textbook and teaching resource

Suggested textbooks

- Marco Bramanti, Carlo D. Pagani, Sandro *Salsa Analisi matematica 1*. Zanichelli 2008.
- Sandro Salsa, Annamaria Squellati, *Esercizi di Analisi matematica.* Zanichelli 2011.

Both of them are available in E-book form.

Lecture Notes available online

- A. Albanese, A. Leaci, D. Pallara, Appunti del Corso di Analisi Matematica 1
- L. De Michele, Appunti di Analisi Matematica 1

Further material

During the course, on the E-Learning platform, the following documents will be made available: the slides of the teaching; many problems and exercises to be solved either under the supervision of the teacher or by yourself; further material on specific topics.

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Semester

First Semester

Assessment method

Written examination with optional oral colloquium.

The goal of the evaluation is to ascertain a correct assimilation of concepts and techniques studied during lessons and exercises sessions. There will be a partial examination (optional) that, if passed, will permit to skip part of the final exam and its score will contribute to the final score.

Written exam

The written exam consists of two part:

- Exercises. The student has to solve exercises similar to those solved in the classroom and/or assigned during the lectures by applying specific computational techniques.
- Theoretical questions. The student is required to state definitions and theorems from the lectures or to illustrate basic concepts and examples taken from the lectures.

The exam is passed if it scores at least 18 points over 30.

Oral exam (optional)

It consists of a discussion that, starting from the exercises of the written exam, can touch different topics of the course.

Partial exam (optional)

During the semester there will be one partial written exam on the topics presented up to that moment (no theoretical questions).

The partial exam is optional but recommanded due to (at least) three reasons:

1. If passed, it allows the students to skip part of the final written examination. The score attributed to the exam will be the average of the scores obtained in the partial and in the final exam.
2. It is useful to verify the student own state of learing
3. If failed, or if the student decides to not use it, it does not affect the score of the final written examination.

Office hours

Office hours is by appointment

Sustainable Development Goals