UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

# SYLLABUS DEL CORSO

## Analisi Matematica I

**2223-1-E4101B001**

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### Learning objectives

The course mainly aims at enabling students to an aware use of basic techniques of infinitesimal (differential as well as integral) calculus for functions of one real variable. The skills gained through the course allow them:

1) to understand a statement concerning contents of the Course and expressed in mathematical terms:

2) to make use of basic tools of differential and integral calculus (limits, derivatives, series ed integrals) for functions of one real variable;

3) to analyze properties of one real variable functions by means of the standard tools provided by the differential and integral calculus (such as asymptotic behaviour, existence of zeros, differentiability, monotonicity and symmetry, extremal properties, namely existence of minimizers/maximizers and their determination, integrability).

### Contents

The contents of the course can be schematically arranged in three strictly intertwined parts:

1) asymptotic estimates (limits of functions useful to establish integrability/summability);

2) differential calculus (first order derivative and beyond) and its applications;

3) series;

4) Riemann integrability of functions and integral calculus.

## Detailed program

1. Numbers and logic: sets and basic concepts of set theory (belonging, inclusion, quantifiers, operations with sets). Sum and binomial coefficeints: sum of a geometric finite sequence, factorials, binomial coefficients and their properties. Newton's binomial formula and Tartaglia's triangle. Basic algebraic properties of real numbers. The order relation over the real number set: boundedness from above and from below, upper/lower bounds, supremum/infimum, maximum/minimum of a subset of real numbers. Continuity axiom of real numbers. Absolute value and its properties.

2. Functions: the notion of function, domain and range of a function. One-to-one property of a function, surjectivity and bijectivity. Power of sets and cardinality: countable and uncontable sets. Composition of functions. Bounded, symmetric, monotone and periodic functions. Graph of a function. Operations with graphs. Invertible functions and inverse. Conditions for the inveribility of a functions. Inverse functions of elementary trigonometric functions.

3. Limits and continuity: sequences of real numbers. Convergent sequences and notion of limit. Uniqueness of the limit and boundedness of convergent sequences.
   Divergent sequences and irregular sequences. Monotone sequences and existence of limit for bounded monotone sequences. Limit of unbounded sequences. Special cases: general behaviour of the geometric and generalized harmonic sequences. Limit and calculus. Limit and inequalities. Infinite and infinitesimal sequences. Asymptotic estimates and comparisons. Quotient criterion. Limit of functions, continuity and asymptotes. Accumulation points. Sequential and topological definition of limit. Limit from left and from right and relationships with the limit. Horizontal, oblique and vertical asymptote. Characterization for oblique asymptotes. Continuity. Continuity of elementary functions. Continuity and operations with functions. Continuity of composite functions. Properties of limits: limit and inequalities; sign persistence phenomenon. Limit and operations with functions. Variable change. Basic limit formulae. Asymptotic relations. Comparisons of infinite and infinitesimal. Hierachies of infinites. Global properties of continuous functions over an interval: theorem about existence of zeros. Global and local maximizers/minimizers of a functions. Weierstrass theorem.

4. Differential calculus for real univariate functions: tangent to the graph of a real value function. Differentiability and first derivative. Equation of the tangent line to the graph of a function. Second derivative and beyond. Derivative of elementary functions. Calculus rules: the derivative under algebraic operations, composition and inverse function. Differentiability implies continuity. Nondifferentiabilities of a function: derivative from the right and from the left, corner points, vertical tangents and cusps. Methods for finding extremals: Fermat theorem and stationarity. Lagrange theorem and monotonicity. Functions with zero derivatives. Theorem of De L'Hospital. Concavity and convexity: convex sets and epigraph of a function. Convexity/concavity for functions. Strictly convex/concave functions. Convexity and tangents. Inflection point. Differential calculus and higher order approximation: the Landau symbol. Polynomial approximations: Mc Laurin and Taylor formulae with remainder in the Peano and Lagrange form.

5. Series: partial sum sequences, character and sum of a series. Geometric series, Mengoli series and harmonic series. A convergence necessary condition. Remainder and remainder series convergence. Criteria for nonnegative series: comparison, asymptotic, root, quotient and Cauchy criterion. Variable sign series: absolute convergence and relationship with plain convergence. Alternate sign series: Leibniz criterion. Sum of series. Taylor series: series expansion. Main Mc Laurin's series (sine, cosine, exponential and log).

6. Integral calculus for one variable functions: integral as a limit of Cauchy-Riemann sum. Geometric interpretation of integral. Properties of integrals: linearity, additivity, positivity and monotonicity. Integral form of the mean value theorem. Primitives of a given function. Fundamental theorem of integral calculus. Primitive of elementary functions. Integration methods: integration by scomposition, variable change and by parts. Integration of rational functions. Generalized integral: integrability criteria in the case of unbounded functions and/or in the case of unbounded integration domains. Two convergence analysis as a reference case. Integrability criteria: comparison criterion and asymptotic criterion. Absolute integrability. Integrability

of the Gaussian function. Integral functions. A remarkable example: the error function. Second fundamental theorem of integral calculus and its consequences. A detailed study of the error function. The Gamma function: definition and basic properties.

## Prerequisites

No inner prerequisite. A refreshement (guided by a tutor, if any) is strongly advised, which should concern the main topics typically taught at the high school. More precisely:

1) algebra: solving algebraic equations of first and second degree, product cancellation and polynomial identity principle;

2) Cartesian geometry: lines, conics (parable, ellipse, hyperbole), exponential and logarithmic functions;

3) trigonometry on the plane: angles in radiants, fundamental trigonometrical functions (sine, cosine and tangent) and related formulae.

## Teaching methods

Class lectures.

Class lectures are aimed at exposing the main ideas behind a notion formulated in mathematical terms and at enabling students to adequately formalize them. In this way, students are enables to read statements about the contents and to apply them for solving problems of various kind.

During the teaching period, some exercise sessions are organized.

## Assessment methods

Students are supposed to pass a written examinaton. For all those students who have passed the written examination, an oral examination is upon request (by the teacher and the student). Interim assessments are also organized once per academic year. For interim assessments, final marks are given as an arithmetic average of the partial scores.

A written examination, both mid-term and complete, aims at certifying the student skills about theoretical contents and calculus techniques provided in the course, as well as their capability in problem solving.
It consists of a test arranged in two main sections: the first one contains closed questions, whereas the second one contains problems/exercises and open questions.
Problems/exercises require to formalize a mathematical question, to apply and combine principles, and to perform computations by means of given calculus tools, while open questions require to expose in detail some theoretical subject (e.g. providing formal definitions, theorem statements, and, whenever requested, proofs, as well as examples and counterexamples) within the course contents.

In the (optional) oral examination students are questioned about all theoretical contents of the course. During the oral examiniation students must be able to argue the solution apporaches proposed in their written test and to discuss in full details the theoretical contents of the course.

If the exam is passed, the final mark is given as sum of the scores obtained in the two parts.

Material for exam simulations is also provided.

## Textbooks and Reading Materials

M. Bramanti, C.D. Pagani, S. Salsa, Analisi Matematica 1, Zanichelli, Bologna, 2008

S. Salsa, A. Squellati, Esercizi di Analisi matematica 1, Zanichelli, Bologna, 2011

A. Guerraggio, Matematica, Pearson, 2014.

Some additional material, in particular anthologies of exercises (with solution and comments) and exam simulations, are provided in the web-page associated to the course.

## Semester

Fall semester.

## Teaching language

Italian

## Sustainable Development Goals

QUALITY EDUCATION