UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

# COURSE SYLLABUS

## Higher Mechanics

**2223-1-F4001Q078**

### Aims

At the end of the course, students will have acquired the following:

- *knowledge*: some of the basic results of the modern theory of Dynamical Systems, Information theory and Algorithmic Complexity;
- *competence* : understanding of the techniques and methods related to the theory together with the ability to deepen specific topic independently;
- *skills*: useful to apply the theory to the investigation of some simple dynamical systems, to solve exercise of increasing level of difficulty, to analyse (even computationally) symbolic sequences of different origin, with particular attention towards applications to biological or literary data.

### Contents

The course aims to provide the student with an in-depth knowledge of the theoretical framework underlying the analysis of symbolic sequences of different origin. The main contents include: statistical approach to dynamical systems, information sources, algorithmic information content.

### Detailed program

The course is divided into three parts:

1. Examples of discrete-time dynamical systems. Elements of topological dynamics. Symbolic dynamics. Ergodic theory. Kolmogorov-Sinai entropy.
2. Shannon entropy. Relative entropy, mutual information. Asymptotic equipartition. Entropy rate for stationary

stochastic processes. Codes: Kraft inequality, optimal codes, efficiency of a code. Universal compressors. LZ78 algorithm.
3. Turing machines. Universal machines. Kolmogorov algorithmic complexity. Universal probability. Halting problem. Chaitin's number. Brudno's theorem.

## Prerequisites

No course of the Master Degree in Mathematics is strictly required for attending the present course. The only prerequisites are the mathematical knowledge, competences and skills acquired during the three-year grade, especially in the courses of Dynamical Systems and Classical Mechanics, Measure Theory, Probability.

## Teaching form

Lectures with blackboard

## Textbook and teaching resource

There is not a single textbook covering all topics.

Many of the topics are covered by:

- M.Brin & G. Stuck, "Introduction to Dynamical Systems", Cambridge University Press. 2002 (1 copia disponibili al prestito in biblioteca; (*e-book online)*
- *P.Walters, "An Introduction to Ergodic Theory", GTM 89, Springer-Verlag (2 copie disponibili al prestito in biblioteca; (e-book not available)*
- T. M. Cover & J. A. Thomas, "*Elements of Information Theory*", 2nd ed., Wiley-Interscience *(2 copie disponibili al prestito in biblioteca; (e-book online)*
- M.Li, P.Vitányi, "An Introduction to Kolmogorov Complexity and Its Applications", second edition, GTCS, Springer-Verlag, 1997; (*e-book online)*

Lecture notes will be distributed covering all the arguments.

## Semester

II Semester.

## Assessment method

Oral exam (of about 45 minutes) in which the student will be assessed both on mathematical aspects of the theory (definitions, statements, proofs), on the application of the theory (examples discussed during lectures), as well as

on the ability to handle the topic independently. Optionally, the student can integrate the exam with the presentation of a project (the choice of the project should be discussed in advance with the instructor). In this case, the relative weight of the project and of the oral examination is equal.

## Office hours

Upon appointment.

## Sustainable Development Goals