

# UNIVERSITÀ DEGLI STUDI DI MILANO-BICOCCA

# **COURSE SYLLABUS**

# **Numerical Mathematics for Machine Learning**

2223-1-F4001Q110

#### **Aims**

In line with the educational objectives of the Master Degree in Mathematics, the course aims to provide the knowledge of the mathematical and numerical methodologies and (available) theories underlying some machine learning techniques.

The techniques and algorithms will be implemented in Python and/or MATLAB, and with the developed codes the students will have the ability to implement elementary algorithms of Machine Learning and to comprehend how more advanced algorithms work.

#### **Contents**

- Mathematical Foundations of "General" Regression Problems
- Classification
- Neural Networks
- Neural Network Learning and Training

# **Detailed program**

#### Mathematical Foundations of "General" Regression Problems

- Review of probability basics;
- Linear regression. Model assessment and selection: Empirical Minimization, Hypothesis Space, Bias-Variance Tradeoff:
- Reproducing Kernel Hilbert Spaces (RKHS), Regularization, Bayesian Interpretation.

## Classification

- Some Numerical Linear Algebra Tools: Singular value decomposition (SVD) and low rank approximation;
- Principal Component Analysis (PCA) and Kernel extension;
- Introduction to Linear Discriminant Analysis (LDA). Non-negative Matrix Factorization (NMF);
- Support Vector Machine (SVM) and Kernel Extension, Image mining, Tree based classifiers, Tree regression.

#### **Neural Networks**

- Motivation and Definition. Mathematical Representation (Neurons, Artificial Neural Networks, Artificial Feedforward Neural Networks)
- Approximation properties, Universal Approximation, Regularity classes, theory-to-practice Generative Adversarial Network

#### **Neural Network Learning and Training**

- Neural Network Learning: Motivation, Regression/Classification;
- Numerical Optimization: Loss functions, back propagation;
- Gradient Descent methods, Stochastic gradient descent methods, accelerated gradient methods, second order methods, constrained optimization and regularization (L2, L1, sparse);
- · Going Deep: Deep Learning. Pros and Cons. Regularization, Convolutional Neural Networks.

#### **CS Laboratory:**

Implementation in MATLAB and/or Python of:

- Feed Forward Neural Networks;
- Back Propagation Algorithm;
- Stochastic gradient descent Applications to artificial and natural images recognition.
  Use of MATLAB and/or Python toolbox for:
- Convolutional Neural Networks

#### **Prerequisites**

Basic courses of the Laurea Triennale.

# **Teaching form**

Lectures (face to face) and CS Laboratory (8 CFU)

#### Textbook and teaching resource

The teaching material will be made available by the instructors during the course.

#### Semester

2?? semester

#### Assessment method

The evaluation of the course has two parts:

- 1- the development of a small project
- 2- an oral exam.

Mark is out of thirty. The student need to reach at least 18/30 in both parts to pass the exam. the final mark is the average of the two partial marks.

The aim of the project is to validate the knowledge and capabilities of the students to use the theoretical and practical instruments developed during the course. Students are encouraged to work on the project **in groups** of at most three people.

The oral exam will evaluate the knowledge and **understanding** of the results and rigorous proofs developed in the course and the capacity to **comprehend** how the algorithms work.

There will be 5 exam sessions (in June, July, September, January, February).

#### Office hours

By appointment

## **Sustainable Development Goals**

**QUALITY EDUCATION**