UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Differential Topology

2223-1-F4001Q111

Aims

The scope of this course is the continuation of the study of Geometry along the path started in the Laurea Triennale (Bachelor). While it is not a strict prerequisite to the other courses in Geometry, which can be taken independently, it aims to unify and connect the different themes and perspectives developed in them.

Differential topology studies the interplay between the differential structure and the topological properties of smooth manifolds. Differential topology is thus also a natural starting base to explore more abstract aspects of algebraic topology. These techniques also yield a geometric approach to intersection theory.

The expected learning outcomes include the following:

- the knowledge and understanding of the basic definitions and statements, as well as of the basic strategies of proof in the theory of differential topology; the knowledge and understanding of some of the most relevant basic applications and examples of the theory;
- the ability to apply the acquired abstract knowledge to the construction and discussion of simple examples and solution of exercises; the ability to expose and communicate effectively and clearly the theoretical content of the course.

Contents

Transversality and intersection theory.
De Rham Theory for smooth manifolds.

Detailed program

- Transversal maps, intersection of transversal varieties.
- Transverality for manifolds with boundary.
- Applications: classifications of 1-manifolds and Brower fixed point Theorem.
- Transversality Theorems.
- Intersection numbers mod 2 and degree of a map mod 2.
- Intersection Theory for oriented varieties.
- Intresection number for oriented varieties, degree of a map.
- De Rham cohomology.
- Poincarè duality on orientable manifolds.
- The Euler class and Euler characteristic.
- Transversal maps, intersection of transversal varieties. Applications: classifications of 1-manifolds and Brower fixed point Theorem. Transversality Theorems. Intersection numbers mod 2 and degree of a map mod 2. Intersection Theory for oriented varieties.

Prerequisites

The content of the courses of Analysis I, Linear Algebra and Geometry, Geometry I. The basics on differential varieties and differential forms (as content of Geometry II and Geometry III). Brief recalls will be offered as needed.

Teaching form

Front lessons at the blackboard.

Textbook and teaching resource

Reference texts:

R. Bott e L. Tu, Differential Forms in Algebraic Topology, Springer-Verlag

V. Guillemin, P. Haine, Differential forms, World Scientific Publishing Co.

V. Guillemin e A. Pollack, Differential Topology, Prentice Hall

J.W. Milnor, Topology from the Differentiable Point of View; University Press of Virginia.

Semester

Second semester

Assessment method

The exam comprises a written test, followed by an oral discussion of the test, which will evaluate the knowledge and understanding of the conceptual framework of the course, as well as the ability to expose it in a well-organized, consistent and effective manner. The test consists of two parts: in the first one there are theorethical questions involving definitions, statement's of theorems, proofs, in the second one computational questions as construction of examples and counterexamples and exercises. The two parts will contribute equally to the the final grade. The evaluation will take into account the exactness of the answers, the clarity of the exposition, the completeness, the rigour, and the mathematical language used.
In order to successfully complete the exam the students need to obtained a grade of at least 18/30.

Office hours

Upon appointment.

Sustainable Development Goals

QUALITY EDUCATION