UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Basic Calculus - 2

2223-1-E1802M115-E1802M108M-T2

Learning objectives

The course aims at giving to the student the basical mathematics tools in order to treat simple mathematical models in economics. In details the aim of the course is to teach to the student real analysis for real functions, with outlines to the calculus in two variables.

Contents

Real functions of real variables and outlines of real functions of two real variables.

Detailed program

UNIT 1 - Real functions of one real variable.
Sets N, Z, Q, R. Sets bounded from above and from belowo; intervals; upper / lower extreme and maximum / minimum of a set.
Definition of function and sequence; calculation of the field of existence; definition of image, image set, reverse image, reverse image set, graph; use of the analytic expression of a function and a sequence. Use of the graph of a function; injective, surjective, bijective functions; functions bopunded from below and from above; lower / upper bound of a function; minimum / maximum, minimum / maximum point of a function; even / odd function; monotonicity of a function and a sequence. Operations with functions, composition, inversion. Simple transformations of graphs. Horizontal / vertical translations, horizontal / vertical reflections; partial horizontal / vertical reflections; rescaling. Composed transformations of graphs.

UNIT 2 - Limits:
Real extended line and neighborhoods; definition of internal, external, border, isolated, accumulation point;

definition of limit of functions and sequences; right / left limit, limit by excess/ defect; reading limits from the graph. Uniqueness of the limit theorem(with dim.), sign permanence theorem (with proof), comparison theorem (with proof). Calculation of limits for functions and sequences.
Continuity. Algebra in extended R, determined forms, limits of exponential and logarithmic functions, arctangent. Indeterminate forms, techniques for solving some indeterminate forms (rational / irrational functions). Asymptotic equivalence and properties. Orders of infinity, hierarchies of infinities.
Negligible function (o-small). Remarkable limits and relative asymptotic equivalences. Indeterminate forms of exponential type and techniques
of solution. Orders of infinitesimal, hierarchy of infinitesimal, o-small. Continuity (from right / left) and discontinuity. Classification of discontinuities. Recognition of discontinuities from the graph and from the analytical expression. Horizontal, vertical, oblique asymptotes. Weierstrass theorem with counterexamples, intermediate value theorem with counterexamples, zero theorem with counterexamples.

UNIT 3 - Derivates:
Incremental ratio and derivative of a function at a point; derivative function; derivatives of elementary functions; calculation of derivatives. Equation of the tangent line; continuity-derivability link, point of inflection to vertical tangent, of cusp, angular. Rule of de L'Hopital; Rolle's theorem (with proof) and counterexamples; Lagrange's theorem (with proof) and counterexamples; derivative of the inverse function. Monotony test (with dim.) And counterexamples; definition of relative extremes; stationary point; Fermat's theorem (with proof); definition of critical point; test of the first derivative for internal extremes. Study of the montonicity of a sequence. Criterion of successive derivatives; test of the first derivative for boundary extremes; definition of concave / convex function; first order test for concavity; second order test for concavity; definition of inflection point.
Taylor and McLaurin polynomials; reminder of Peano; use of the Taylor polynomial for the computation of limits.

UNIT 4 -Complete study of a functions and functions of two variables
General scheme for the study of function. Analytic and graphical domains for real functions of two real variables; level curves; partial derivatives, gradient, stationary points.

Prerequisites

Set theory. Powers, logarithms, exponentials and their properties.
First and second degree inequalities, rational inequalities, logarithmic and exponentials inequalities. Cartesian equations of the line, of the circumference, of the parabola, equation of straight line passing through two points. Basics of trigonometry.

Teaching methods

Lectures (in attendance unless otherwise required by the university) divided into:

- theoretical lectures, focused on the knowledge of definitions, results, proofs and relevant examples
- exercise classes, focused on the skills necessary to apply the theoretical knowledge and competencies to the solution of exercises.

Assessment methods

Written exam with 5 exercises and 3 theory questions. The outline of the exercises is as follows:

Exercise 1: Transformations of graphs of elementary functions;
Exercise 2: Limits;
Exercise 3: Various;
Exercise 4: Two-variable functions;
Exercise 5: Full Function Study.
The written test evaluates the formal correctness of the passages, the adequacy of the mathematical language adopted, the skills and knowledge acquired during the course.
Once the written exam has been passed, the professor or student can request a supplementary oral exam. The oral exam focuses on the entire program of the course and can contribute both positively and negatively to the final grade.
The course does not include the splitting of the exam into intermediate tests.

Textbooks and Reading Materials

Textbooks

Torriero, A., Scovenna M., Scaglianti, L.: Manuale di matematica. Metodi e applicazioni. CEDAM
Scovenna, M., Grassi, R.: Matematica – Esercizi e temi d'esame. CEDAM.

Additional texts to which reference may be made

A. Guerraggio, (2014): Matematica. Prentice Hall, second edition.
G. Monti, R. Pini: Lezioni di matematica generale: funzioni reali di variabile reale, L.E.D.

Additional teaching material

Course notes and teaching material provided on the e-learning platform

Semester

First semester, first year

Teaching language

Italian

Sustainable Development Goals