UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Analisi Complessa

2324-3-E3501Q057

Aims

The aim of the course is to make students able to effectively use the powerful methods of complex analysis in theoretic and practical applications.

Specifically, the expected learning outcomes include:

- the knowledge and understanding of the fundamental definitions and statements, as well as of the basic strategies of proof in Complex Analysis;
- the skill to apply such conceptual background to the construction of concrete examples and to the solution of exercises, ranging from routine to challenging (starting with routine exercise that require straightforward application of the definitions and the results given during the lectures, up to exercise that require deep understanding of the matter and the ability of developing original ideas).

Contents

This is a basic course in one complex variable. It includes holomorphic functions, power series, Cauchy's theorem and applications, isolated singularities, zeroes of entire functions and applications. We shall also provide an introduction to the Riemann zeta function and the Riemann conjecture, the most famous amongst the conjectures in Mathematics.

Detailed program

Part 1. Preliminaries. Holomorphic functions: definition and examples. Entire functions. Holomorphic functions and diffrentiable maps on R 2. Cauchy–Riemann equations. Power series. Hadamard's formula for the radius of convergence. Series expansions of e z , sin z and cos z. Power series define holomorphic function in the disc of convergence. Integration along curves. Parametrized curves, smooth curves, and piecewise smooth curves. Properties of integration along curves. Primitive of a function and its properties. Functions with anishing first derivative are constant in regions,

Part 2. Cauchy's theorem and applications. Goursat's lemma. Local primitives and the Cauchy theorem for discs. Extensions to toy contours. Computations of integrals. Examples. Cauchy's integral foolomorphic functions are locally sums of power series. Liouville's teorem. Fundamental theorem of algebra. Analytic continuation and identity principle for holomorphic functions. Morera's theorem. Uniform convergence on compacta of sequences of holomorphic functions. The symmetry principle and Schwarz's reflection principle. Runge's theorem.

Parte 3. Meromorphic functions and the logarithm. Zeroes and poles. Residues and the residue formula. Isolated singularities of holomorphic functions, and Riemann's theorem on removable singularities. Poles and essential singularities. The Casorati–Weierstrass theorem. Singularities at infinity. Characterisation of meromorphic functions on Riemann's sphere. The argument principle. Rouche's theorem. The open mapping and the maximum modulus theorems. Homotopic paths, and the general form of Cauchy's theorem. The logarithm. Existence of the logarithm in simply connected domains, and related properties.

Parte 4. Entire functions. Jensen's formula. Functions of finite order. Entire functions and its zeroes. Infinite products. Criterion of convergence. Infinite products of holomorphic functions. Product formula for sin. Weierstrass' canonical products. Entire functions with prescribed zeroes. Hadamard's factorization theorem. Factorization of entire functions of finite order.

Parte 5. Euler's Gamma function and its properties. The Riemann's zeta function and its analytic continuation. Introduction to Riemann's conjecture.

Prerequisites

The prerequisites are included in the programme of the courses Analisi I, Analisi II, Algebra lineare and Teoria della misura of the Laurea triennale in Matematica. Specifically we require a sound knowledge of differential and integral calculus in one and several variables, basic notions in Linear algebra and a good understanding of the Lebesgue integral, in particular of the Lebesgue dominated convergence Theorem and the Fubini-Tonelli Theorem.

Students lacking prerequisites are invited to contact the professor by e-mail. He will give them bibliographical suggestions useful to fill the gaps and possibly provide further support.

Teaching form

Under "normal circumstances" the lecture will be held in the lecture hall with blackboard.

Textbook and teaching resource

Stein and Shakarchi, “Complex analysis”, Princeton University Press.

Exercises, made available on the e-learning page of the course.

Semester

I semester

Assessment method

There will be no mid-term exams.
Written examination, including theoretical questions (proofs of part of the results illustrated during the course) and exercises, often similar to those solved during the class hours. In order to get a positive grade, both the parts including theoretical questions and exercises must get a passing grade.

The grade will take into account the exactness of the answers, the clarity of the exposition and the command of mathematical language used.

Office hours

Upon appointment.

Sustainable Development Goals