UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

# COURSE SYLLABUS

# Geometry III

**2324-3-E3501Q055**

## Aims

This course aims to yield an introduction to the fundamental group and differentiable manifolds.

Expected learning outcomes include:

- knowledge of definitions and basic results on the fundamental group and differentiable manifolds, knowledge of important examples;
- comprehension of the main proof techniques;
- ability to solve problems and discuss the geometric properties of concrete examples.

## Contents

Fundamental group and covering spaces of a topological space. Differentiable manifolds: topological manifolds, differentiable structures, tangent space, tangent bundle, calculus on manifolds. Immersions, submersions, submanifolds. Vector fields, flows, Lie bracket. Differential forms, Stokes' theorem, De Rham cohomology.

## Detailed program

- Fundamental group of a pointed topological space. Definition and elementary examples. Homomorphism induced by continuous functions. Base-point dependence and arcwise connectedness. Fundamental group of the sphere. Homotopy of continuous functions and induced homomorphism. Homotopy invariance of the fundamental group. Presentation of groups and free products. The Seifert - Van Kampen theorem and its applications. Coverings.

- Topological and smooth manifolds: definitions and elementary examples. Smooth functions between manifolds. Diffeomorphisms. Tangent space. Differential of a smooth function between manifolds. Immersions, embeddings, submersions. Regular submanifolds. Tangent bundle and vector fields. Integral curves and flow of a vector field. The Poincaré-Hopf theorem. Lie bracket. Lie algebra structure on the space of vector fields. Lie derivative of a vector field. Cotangent space and cotangent bundle. One-forms. Multilinear forms and wedge product between them. Differential forms. Exterior algebra of a smooth manifolds. Exterior derivative. Closedness and exactness. De Rham cohomology. Poincaré Lemma. De Rham Theorem. Partitions of unity. Integration of differential forms on a manifold. The Stokes' theorem and its applications.

## Prerequisites

The content of the courses of Geometry I and II, Mathematical Analysis I and (in part) II, Linear Algebra and Geometry.

## Teaching form

Blackboard lectures.

## Textbook and teaching resource

### Textbooks

1st part: - W. Fulton - Algebraic Topology. A first course - Springer, 1995*

- C. Kosniowski - A first course in Algebraic topology - Cambridge University Press, 1980**

2nd part: - L. W. Tu - An Introduction to Manifolds (Second Edition) - Springer, 2011*

### Further readings

- L. I. Nicolaescu - Lectures on the Goemetry of Manifolds (Second Edition) - World Scientific, 2007*
- J. M. Lee - Introduction to Smooth Manifolds - Springer, 2012*
- V. Guillemin and P. Haine - Differential forms - World Scientific, 2019

**This book is available in electronic format at the UNIMIB library webpage.*

## Semester

First semester.

## Assessment method

Written and oral examination. Homework during the class period (not mandatory) for getting exemption from the written part.

## Office hours

By appointment.

## Sustainable Development Goals

QUALITY EDUCATION