UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Mathematical Analysis II

2324-2-E3501Q008

Aims

A basic course in integral and differential calculus in several variables and ordinary differential equation

Expected learning outcomes include

Knowledge: acquiring the notion of Banach space with some classic examples in mind:
continuous / limited functions on a compact inetrval. Understanding of the definitions and main
results of the differential calculus in several variables and of the theory of ordinary differential equations

Capacity: acquire the main integration techniques for functions of several variables in domains delimited by regular curves as well as the ability to apply the aforementioned abstract knowledge to concrete problems.

Contents

Complete metric spaces and Banach spaces: examples. Sequences and series of functions. Directional derivatives, differentiable functions, higher order derivatives, critical points and local extrema. Integral calculus in several variables: Fubini's theorem; change of variables; polar, spherical and cilindrical coordinates. Ordinary differential equations: existence, uniqueness and continuous dependence on initial data. Implicit function theorem and Lagrange multipliers.

Detailed program

1. Differential calculus in more than one variable.

- Directional derivatives. Differentiable functions. Link between the directional derivatives in the case of differentiable functions. Bonds between continuity and derivability and differentiability. Higer order derivatives. Derivatives of compound functions.
- Maximum and minimum in open sets: necessary condition for differentiable functions and level curves.
- Positive defined/semidefinite matrices and related Matrix criteria Hessian. Taylor's formula arrested on the second order. Convexity and related criteria for the recognition of theirs extrema.
- Recognition of the maximums and minima by the Hessian matrix

2. Integral calculus in more than one variable.

- Definition of Riemann integral for a real valued function of several variables.
- Measurable sets according to Peano-Jordan: necessary and sufficient condition for measurability in the case of bounded sets. Examples of measurable and non-measurable sets.
- Reduction method (Fubini's theorem) for multiple integrals. The center of gravity of a two-dimensional measurable set. Solid volume.
- Change of variables in double and triple integrals (*): polar, spherical and cylindrical coordinates. Volume of rotation solids: Guldino's theorem.
- Improper integrals.

3. Curves and surfaces.

- Regular / piecewise regular curves in R?. Length of a curve: equivalent definitions and independence from the parametrisation (proof of the Theorem 15.4 of the book of Giusti is not required). Curvilinear abscissa
- Theorem of implicit functions (Dini) in the two-dimensional case. Cartesian surfaces; sufficient conditions for one surface to be locally a chart. Vector product and theorem of Dini in the case of a function from R^3 to R^2 (without proof)
- Maximum and minimum in compact sets: Lagrange multipliers (proof only in the case of a constraint in R^2).

4. Sequences and series of functions

- Normed vector spaces: examples (C ([a, b]), B (I) (with I interval) C?([a, b]). Banach spaces.
- Theorem of contractions.
- Punctual and uniform convergence for sequences / series of functions. Weierstrass criterion for the series of functions. Uniform convergence and boundedness/ continuity of the limit function. Pointwise / uniform convergence for derivatives of functions. Limits of sequences of integrable functions.
- Power series: convergence radius. Taylor series. approximation of integrals through the use of power series.

5. Differential equations

- Cauchy problem for first order equations. Theorem of existence and local uniqueness. Extension of solutions. Maximal solution and related properties (*). Sublinearity condition. Theorem of existence and global uniqueness (proof is not required in the sublinear case).
- Equations of order n: equivalence with a system of the first order. Linear systems of the first order. Structure of the space of the solutions of a homogeneous system of the first order. Solutions in the non-homogeneous case. The Variation of constants method in the case of an equation of order n.
- Exponential matrix: definition and properties. Calculation of the exponential matrix in the case where the matrix is ??diagonalizable on the real field. The calculation is required in the other cases only for 3 x 3 matrices.
- Solution methods for some particular equations and systems.

Prerequisites

Analysis I, Linear Algebra, Geometry I

Teaching form

lessons (8cfu), exercises (4cfu) and tutoring. Lessons in Italian

Textbook and teaching resource

Textbook: C.Pagani; S.Salsa: Analisi Matematica 2 Ed. Zanichelli

Other teaching resources

Enrico Giusti: Analisi Matematica II ed. Bollati Boringhieri.

A. Bacciotti; F. Ricci: Lezioni di Analisi Matematica 2 Ed. Levrotto & Bella /Torino

C.Pagani; S.Salsa: Analisi Matematica 1 Ed. Zanichelli

Enrico Giusti: Analisi Matematica 2, old edition, Bollati Boringhieri

Semester

First semester.

Assessment method

Oral and written exam: weighted 1/2 and 1/2 of the final mark
Written test:
The maximum mark of the written test is 33/33.
The written test consists of open questions; for each of them the following will be evaluated: consistency, in particular contradictory statements relating to the same question will be penalised; the acquisition of techniques illustrated in lessons and/or exercises; the ability to apply the theoretical contents of the course to problem solving.

Students with a score lower than 16/33 will not be admitted to the oral exam.

Oral test.
The oral test begins with a discussion of the written test: students may be asked to discuss the mistakes made or to carry out an exercise of the same type as those not addressed in the written test. Finally, students will be asked to present the statements and proofs of theorems, definitions, possibly in written form. Examples/counterexamples and calculation techniques introduced and covered in class and/or exercise must be known.

During the year there are 5 exam sessions in the following periods: two distributed between the monthsin January and February, one in June, one in July and one in September. Each exam session first includes a written test and then, if the written test is passed, a theoretical/oral test a few days later.

Two partial written tests will be held during the period of the lessons. Each of them has a maximum score of 33 and it will cover only half of the program and will consist of open questions. To access the second partial test it will be necessary to have obtained a score of not less than 14/33 in the first one.To access them, the student must have passed the Analysis I exam. The intermediate tests are reserved for those who have the Analysis II exam for 2023-24 in their study plan.

The final mark of the intermediate tests is the average of the marks of both.
Those who have obtained a final grade greater than or equal to 16/33 in the intermediate tests will be able to directly access one of the oral exams in January or February (at their choice).

Office hours

By appointment.

Sustainable Development Goals

GENDER EQUALITY