UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Advanced Foundations of Mathematics for Ai

2324-1-F9102Q001

Aims

The aim of this course is to provide the mathematical foundations for the optimization and data manipulation algorithms, which are widely employed in the artificial intelligence domain. A number of applications are provided, so that the student will be able to tackle and solve constrained and unconstrained optimization problems, analyze data through dimensionality reduction and Fourier transforms, while the theoretical part will constitute a solid background for understanding and mastering related, more recent techniques, which are constantly developed in the field.

Contents

The course consists of a theoretical part and a part of exercises. The theoretical part will start by recalling the basic concepts of linear algebra and multivariate calculus which are needed, then will cover the topics of optimization (in particular, convex optimization), dimensionality reduction techniques and Fourier transforms. In the exercises examples of related problems and applications are given.

Detailed program

- Linear Algebra: Eigenvalues, eigenvectors, diagonalization and spectral theorem. Positive definite matrices, singular value decomposition.
- Fourier transform and series: Fourier series for periodic functions, Fourier transform of continuous and discrete signals. Definitions and basic properties, inversion and differentiation, convolutions.
- Multivariate calculus: Partial derivatives, differential, Jacobian matrix, Hessian matrix, Taylor’s theorem.
- Optimization: Unconstrained critical points and characterization through Hessian matrix. Gradient descent, Newton’s method. Implicit function theorem, constrained critical points, Lagrange multipliers.

- Convex optimization: Convex sets, convex functions. Convex conjugates.
- Convex optimization problems: definition, notable cases, duality, strong duality and optimality condition.
- Linear and nonlinear dimensionality reduction techniques: Linear projectors, principal component analysis, independent component analysis, kernel principal component analysis.

Prerequisites

Foundations of calculus: derivatives, integrals, numerical series. Foundations of linear algebra: vector spaces and linear applications, matrix representation.

Teaching form

Lectures and assisted exercises. Both of them will be held in presence, unless further COVID-19 related restrictions are imposed, and the attendance is highly recommended.

Textbook and teaching resource

M. P. Deisenroth, A. A. Faisal, C. S. Ong, Mathematics for Machine Learning, Cambridge University Press (2020). Lecture notes.

Semester

First.

Assessment method

The exam is individual and consists of a written and an oral part. In the written exam, the proficiency in applying mathematical notions to the solution of exercises and problems is evaluated. As an alternative to the written exam, two partial written exams are planned. The oral exam is focused, instead, on assessing the knowledge of the mathematical notions and the ability of expressing them in an adequate way, as well as on establishing the understanding of the deduction processes which link the mathematical objects.

Office hours

By email appointment.
Office: 3022, University of Milan-Bicocca, Department of Mathematics and Applications, Via Roberto Cozzi 55 - 20125 MILANO
Building U5-Ratio

Email: alberto.maiocchi@unimib.it

Sustainable Development Goals

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