UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Algebra Superiore

2324-1-F4001Q112

Aims

In line with the aims of the CdS, the course will provide students the knowhow necessary to deal with transmission of information via noicy channels, in order to analyze optimal error-correcting and -detecting procedures. Time permitting some rudiments of programming languages as Magma and Gap will be imparted. These tools serve to emphasize sperimental aspects of mathematical discovery. We will also impart the necessary skills to comprehend and analyze the main technical and proof methods.

Those will be tested via problem solving and resolution of exercises related to the contents of the course.

Contents

1. Recalls on cyclic and abelian groups, normal subgroups, quotient groups, simple groups;
2. Holder program, composition series, direct, semidirect and wreath products, extension, cohomology;
3. Commutators, derived subgroups, nilpotent and solvable groups, Sylow and Hall subgroups;
4. Generators and Relations, free groups, Schreier theorem;
5. Permutation groups, transitivity, primitivity, O'Nan-Scott theorem;
6. Classification of finite simple groups, semisimple Lie algebras, Chevalley bases, finite groups of Lie type.

Detailed program

1. Recalls on the classification of cyclic and finitely generated abelian groups, conjugationon elements and subgroups, normal subgroups, quotient groups, simple groups, example: alternating groups, simplicity of Alt(5), projective special linear groups;
2. Holder program, composition and chief series, internal and external direct product, automorphisms,

semidirect product, extensions, coborders, cocycles, first and second cohomology group and thier construction, Schur-Zassenhaus theorem;
3. Commutators, Hall-Witt theorem, Hall-Petrescu theorem, derived, lower and upper-central series, nilpotent groups, solvable groups, simplicitiy of alternating groups, irresolubility of equations of degree al least 5, Sylow and Hall subgroups, characterization of soluble groups;
4. Generators and relations, free and finitely generated/presented groups, Schreier-Nielsen theorem, subgroups of free groups;
5. Permutation groups, transitivity, primitivity, maximal subgroups, Aschbacher theorem, O'Nan-Scott theorem;
6. Alternating groups, special linear, orthogonal, unitary and symplectic projective groups, Steinberg, Ree, Suzuki groups,sporadic groups, semisimple Lie algebras, Chevalley bases, constructing finite analog of Lie groups.

Prerequisites

`Algebra 1 and 2`

Teaching form

The course consists of Lectures for 8 credits. They will give knowledge of basic definitions, relevant results and theorems. On the other side, we intend to give skills to use results and knowledge in solving exercises and analysing problems.

The official language is italian but under request it will be switched to english.

Textbook and teaching resource

**Textbooks:

- Machi', Gruppi, Springer Verlag 2012
- Kurzweil, Stellmacher, The theory of finite groups, Springer Verlag 2004
- Robinson, A course in the theory of groups 2ed, Springer Verlag 1996
- Appunti videoscritti delle singole lezioni reperibili su questa piattaforma.
- Appunti scritti in LaTeX in formato pdf reperibili su questa piattaforma.

Further Readings:

- Aschbacher, Finite Group Theory 2ed, CUP 2000
- Carter, Simple groups of Lie type, Wiley & Sons, 1989
- Humphreys, Introduction to Lie algebras and representation theory, Springer-Verlag 1972
- Sambale, Endliche Permutationsgruppen, Springer Spektrum, 2017

Semester

First semester.

Assessment method

The exam consists of an oral enquiry assessing both the student's acquisition of the course contents and her/his capabilities of analyzing and solving problems. Both aspects equally contribute to the final score.

Mark range:18-30/30.

Office hours

By appointment.

Sustainable Development Goals

QUALITY EDUCATION