COURSE SYLLABUS

Complex Geometry

2324-1-F4001Q069

Aims

The aim of the course is to introduce students to the basic aspects of Complex Geometry, with a special emphasis on its deep relation to the other branches of Geometry, as well as to Analysis and Algebra.

The expected learning outcomes include the following:

- the knowledge and understanding of the basic definitions and statements, as well as of the basic strategies of proof in the theory in differential geometry; the knowledge and understanding of some of the key foundational examples of the theory;
- the ability to apply the acquired abstract knowledge to the solution of simple computational exercises and theoretical problems, referring in a precise and well-organized manner to the pertinent results being used;
- the ability to apply the theoretical background to the construction and discussion of simple examples and solution of exercises;
- the ability to expose and communicate effectively and clearly the theoretical content of the course.

Contents

The first part of the course will deal with the theory of Riemann surfaces, that is, surfaces endowed with a complex structure. The second will have a more algebraic cut, and will be a basic introduction to the language and techniques of complex Algebraic Geometry.

Detailed program

- Generalities on complex structures.

- Topology of Riemann surfaces.

- Holomorphic maps, branched coverings

- The Riemann surface of an algebraic curve

- Riemann-Hurwitz theorem

- Holomorphic and meromorphic forms

- Riemann bilinear relations

- Divisors and linear series, Theorem of Riemann-Roch

- Basics of Commutative Algebra

- Localization

- Normalizations

- Hilbert's Nullstellensatz

- Affine varieties and their morphisms

- Projective and quasi-projective varieties

- Notable examples (Algebraic groups...)

Prerequisites

The content of the courses in Algebra, Analysis, and Geometry of the first two years of the Laurea Triennale in Mathematics. In addition, the courses in the same subjects in the third year of the Laurea Triennale are highly recommendedi for an optimal preparation to this course.

Teaching form

Lectures at the blackboard. The first part will be taught in Italian, unless the presence of foreign students makes the use of English preferable. The second part will be taught in English irrespective of the composition of the class.

Textbook and teaching resource

- Registered lessons
- Recommended textbooks:

1. W. Fulton Algebraic Topology A first course Springer Verlag

2. P. Griffiths Introduction to Algebraic Curves AMS

3. I. R. Shafarevich Basic Algebraic Geometry I Springer Verlag

Semester

first semester

Assessment method

During the course, two written partial tests will be offered, each referred to one half of the course. Each partial test will consist of a balanced flexible combination of computational exercises and theoretical questions. The exercises and theoretical questions in these tests will be along the lines of those offered in the practical and theoretical tests of the regular exam sessions (see below). The two partial tests will contribute equally to the final grade. To pass the exam through the partial tests, the student needs to pass each of them, thus obtaining a grade of at least 18/30 in both.

Alternatively, students may pass the exam through the regular exam sessions that follow the end of the course, and exactly the same pattern will be offered in every exam session. Thus, each session comprises two written tests, each referred to one half of the course, and consisting of a balanced combination of computational exercises and theoretical questions. The theoretical questions will involve definitions, statements of theorems, proofs, construction of examples and counterexamples, and simple theoretical problems.

The exercises will measure the student's ability to master the acquired formalism and apply it to some simple computations, to build on the acquired theoretical knowledge, and to invoke it in a pertinent and precise manner.

The theoretical questions will evaluate the knowledge and understanding of the conceptual framework of the course, as well as the ability to expose it in a well-organized, consistent and effective manner.

In order to pass the exam in one of the regular sessions, the student needs to obtain a grade of at least 18/30 in each of the two tests, which will contribute equally to the final grade. The two tests needn't be undertaken in the same session. It is also allowed to pass one the tests during the course and the other in a regular exam session.

To each exercise/theoretical question (or problem) a maximum partial grade will be assigned by the commission, depending on its difficulty and length. In the evaluation, every student will be given a grade in correspondence to each exercise/theoretical question (or problem) up to the maximum one, measuring the exactness, the completeness, the rigour, the clarity and the overall coherence of the development.

The exact subdivision of the course in two parts will be communicated well in advance during its duration.

Office hours

upon appointment

Sustainable Development Goals

QUALITY EDUCATION