UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Differential Geometry

2324-1-F4001Q071

Aims

The aim of the course is to introduce the foundations of the theory of Riemannian manifolds, that is, manifolds endowed with a Riemannian metric, which consists in the assignment to each tangent space of a smoothly varying Euclidean product. The students will familiarize with the most basic concepts and techniques of differential geometry, moving from the foundational concept of Levi-Civita connection. Starting from the latter, the basic local curvature invariants and the notion of geodesic will be introduced. A key aspect which we propose to illustrate is the interplay between local aspects of the Riemannian metric, and the global topological structure of the underlying manifold.

At the end of the course students are expected to have acquired:

1. the basic notions and results of the classical Riemannian Geometry.
2. the abilty of verifying on concrete examples the main geometric properties of Riemannian manifolds.
3. the ability to process what was seen in class and to independently proceed with the study of advanced aspects of the theory by completing the details of the various arguments.

Contents

Starting from the problem of the existence of a Riemannian metric on a generic differential manifold, we will move on to the notion of Levi-Civita connection, and of corresponding parallel transport, which will allow us to define the concept of geodesic curve as a curve with zero acceleration. The study of the Riemann curvature tensor and its traces, also concretized in the case of regular surfaces in Euclidean space, will precede the culminating part of the course which, time permitting, will be dedicated to some global aspects such as the characterization of completeness according to the theorem of Hopf-Rinow and the link between the sign of curvature and the topology of a complete Riemannian manifold.

Detailed program

1. Outline of regular surfaces in Euclidean space and their curvatures
2. Definition and existence of Riemannian metrics
3. Levi-Civita connection and parallel transport
4. Geodesics and expontential map
5. Jacobi fields and conjugate points
6. The intrinsic metric structure of a Riemannian manifold
7. Curvatures of a Riemannian manifold
8. Global results (time permitting)
   8.1) Global theory of geodesics and completeness
   8.2) The Bonnet-Myers and Cartan-Hadamard theorems

Prerequisites

Differential calculus in several variables, basic notions of differentiable manifolds, linear and multilinear algebra.

Teaching form

Lectures, exercises to be carried out at home and individual study of an advanced topic not exposed in class.

The course is scheduled in Italian but could be held in English in the presence of foreign students

Textbook and teaching resource

Basic textbooks

M. P. do Carmo *Riemannian geometry*. Birkhäuser Boston, Inc., Boston, MA, 1992.

Lee, John M. *Introduction to Riemannian manifolds*. Second edition. Graduate Texts in Mathematics, 176. Springer, Cham, 2018.

Tu, Loring W. *Differential Geometry*. Graduate Texts in Mathematics, 275.

Additional teaching material (such as lecture notes) will be provided during the course

More advanced textbooks

S. Gallot, D. Hulin, J. Lafontaine *Riemannian geometry*. Third edition. Universitext. Springer-Verlag, Berlin, 2004.

P. Petersen *Riemannian Geometry*. Graduate Texts in Mathematics, 171. Springer, 2006.

Semester

Second semester

Assessment method

The verification of learning will consist of two parts:

1. A traditional oral exam, during which the student will have to show that she/he has acquired the basic notions, the demonstrations of the main theorems, and the ability to analyze and calculate on some concrete examples.

2. A seminar with which the student must demonstrate that she/he has acquired the ability to individually understand an advanced topic not exposed in class, completing all the details of the arguments.

Office hours

By appointment

Sustainable Development Goals

QUALITY EDUCATION