UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Analisi Funzionale

2324-1-F4001Q075

Aims

Consistently with the educational objectives of the master degree in mathematics, the course aims to provide students with the knowledge about the definitions and the basic theorems of functional analysis. The skills needed to understand and analyze the main techniques and demonstration methods related to the theory, and the skills to apply them to face problems in different areas of Mathematics will also be trained. Particular emphasis will be placed on problem solving.

Contents

Locally compact Hausdorff spaces. Spaces of continuous functions. L? Spaces. Compactness in C? and in L?. Weak and weak* (weak star) topologies. Compactness in the weak topologies. Riesz representation theorems.

Detailed program

Metric spaces, normed vector spaces, compactness of the closed ball and dimension.

Spaces of continuous functions. Urysohn Lemma and cut-offs. The Stone-Weierstrass theorem: density and separability. Compactness in the spaces of continuous functions: the Ascoli-Arzelà Theorem.

L? spaces and their basic properties. Density and separability: the Lusin Theorem. Compactness in the L? spaces: the Kolmogorov-Riesz Theorem. The canonical embedding of Frechet-Kuratowski for separable metric spaces.

Linear functionals and weak topology on a normed space. Sub-additive positively homogenous functionals. The Hahn-Banach theorem: general form. Convexity and hyperplane separation. Mazur Theorem: weak and strong closure of convex sets.

The weak* (weak star) topology. Bi-dual and the James embedding. The Banach-Alaoglu Theorem: weak* compactness of the closed ball in the dual space.

Reflexive spaces. Reflexivity in the L? spaces. Uniform convexity and reflexivity. Weak and strong convergence in uniformly convex spaces. Kakutani and Eberlein-Smulyan theorems: weak compactness of the closed ball and reflexivity. Sequential compactness in the weak* topology.

Locally convex topological vector spaces. Convex hull and extremal points: the Krein-Milman theorem.

Signed measures. The Radon-Nikodym Theorem. Duality in the spaces of continuous functions: the Riesz Representation Theorem.

Prerequisites

Elements of the theory of abstract integration, elements of L? space theory , elements of general topology. Basic knowledge of Banach spaces and Hilbert spaces. Basic problem-solving skills.

Teaching form

Lectures are organized to introduce the main theoretical concepts, to present the main ideas of the proofs of the theorems and to analyze explicit examples/problems. Take-home exercises will be assigned in order to train the problem solving skills of the students, and to elaborate on some aspects of the theory.

The lectures are in Italian, unless there are non-Italian speakers among the students in which case they could be in English.

Textbook and teaching resource

Bibliographic references

- H. Brezis. Functional analysis, Sobolev spaces and partial differential equations. Universitext. Springer, New York, 2011.
- G.B. Folland. Real analysis. Modern techniques and their applications. A Wiley-Interscience Publication. John Wiley & Sons, Inc., New York, 1999.
- W. Rudin. Real and complex analysis. McGraw-Hill Book Co., New York, third edition, 1987
- T. Bühler and D. A. Salamon. Functional analysis. Volume 191 of Graduate Studies in Mathematics. AMS, Providence, RI, 2018.

Further material

On the e-learning page of the course, the following will be made available:

- Some lecture notes, or links to online resources
- Exercises and problems.

Semester

First semester.

Assessment method

The final exam is written, with the possibility of an oral part. There are no partial exams during the course. The written exam consists in the resolution of exercises/problems, with the aim of testing the knowledge of the students on the topics of the course, as well as testing their problem-solving skills.

The oral part of the exam is not mandatory, but it can be asked for by the student or the teacher. It consists of a discussion about the written part of the exam, and the resolution of other exercises/problems. Without an oral exam, it is not possible to obtain a mark greater than or equal to 28.

Office hours

By appointment (to be scheduled via e-mail).

Sustainable Development Goals

QUALITY EDUCATION