UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Geometric Group Theory

2324-1-F4001Q082

Aims

The main goal of this course is the study of certain graphs called trees and of the groups acting on them. This theory was developed by Hyman Bass and Jean-Pierre Serre in the ‘70s and is now a building block in the study of Geometric Group Theory, which aims to recover group properties by observing how groups act on certain topological spaces. Time permitting, we will see some application of the theory, such as ends of groups, the characterization of subgroups of free groups via Stallings graphs, the boundary of a tree, etc.

- *knowledge:* learning the language, definitions and statements of the main results of Bass-Serre theory;
- *expertise:* concrete understanding of the main proof techniques;
- *skills:* ability to apply the theory to solve problems

Classes will be taught in English.

Contents

Graphs, trees, Cayley graphs; actions of groups on graphs, free groups, free products and their generalizations

Detailed program

- graphs, paths, connectivity, trees.
- group actions on graphs, Cayley graphs, quotient graphs;
- free groups, diamond and ping-pong lemmas
- free products (with amalgamation), HNN-extensions.
- graph of groups; the fundamental group of a graph of groups;

- Characterization theorem for groups acting on trees (the fundamental theorem of Bass-Serre theory)
- Kurosh theorem for subgroups of free products

Prerequisites

Algebra I, Geometria I.

Teaching form

Face-to-face education

Textbook and teaching resource

- O. Bogopolski, Introduction to Group Theory, EMS Textbooks in Mathematics, 2008.
- J. Meier, Groups, Graphs and Trees, London Mathematical Society, Student Texts, 73, CUP, 2008.
- J-P. Serre, Trees, Springer-Verlag, Berlin, 1980.
- W. Dicks, M. Dunwoody. Groups acting on graphs. Cambridge Studies in Advanced Mathematics, 17. Cambridge University Press, Cambridge, 1989
- G. Baumslag, Topics in combinatorial group theory. Lectures in Mathematics ETH Zürich. Birkhäuser Verlag, Basel, 1993

Semester

1st semester

Assessment method

A 20 minutes presentation on an application of or a topic related to Bass-Serre theory previously agreed with the instructor, followed by an oral examination on the content of the course to assess the students' acquired proficiency in explaining and applying Bass-Serre theory. The talk will contribute 20% to the final grade, while the oral exam will provide the remaining 80%.

Office hours

By appointment.

Sustainable Development Goals

QUALITY EDUCATION