UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Numerical Linear Algebra

2324-1-F4001Q119

Aims

The main goals of the course are:

- Knowledge (and understanding) of the Singular Value Decomposition and its wide uses.
- Knowledge (and understanding) of state-of-the art algorithms for eigenvalue computation
  of core algorithms for solving linear systems, including in particular iterative solution methods of Krylov subspace type
- Knowledge (and understanding) of some of the techniques for constructing and analyzing randomised algorithms.
- Ability to construct and analyse the numerical algorithm. Ability to interpret and analyse numerical results

Contents

This course builds on elementary linear algebra and in it we derive, describe and analyse a number of widely used constructive methods (algorithms) for various problems involving matrices. Numerical Methods for solving linear systems of equations, computing eigenvalues and singular values and various related problems involving matrices will be the main focus of this course.

In the second part of the course we will introduce some basic techniques for designing and analzying randomized algorithms.

We shall cover the subject from mathematical point of view, studying how to construct modern computational algorithms, exploring their properties and validating the algorithms in concrete problems.

Detailed program

0- Introduction. Recap on Linear Algebra concepts and basic matrix decompositions.
1- SVD and its properties. Applications of SVD. Truncated SVD
2- Direct methods for linear systems and least-squares problems
3- Stability. Perturbation Theory. Backward Stability & error analysis
4- Direct methods for eigenvalue problems

5- Krylov subspace methods (for eigenvalue problems and linear systems)

6- Randomised algorithms (for linear systems & least squares)

7- Other Applications of SVD. Clustering and Nonnegative Matrix Factorization. Further Applications in Data Mining and Pattern recognition.

Prerequisites

Solid knowledge of Analysis, Linear Algebra and basic Numerical Analysis.
Solid knowledge of MATLAB
Desiderable: Good knowledge of basic probability

Teaching form

Lectures in class and in the Lab.

We will use MATLAB for all computer examples, exercises and projects.

The students will be given the possibility of adhering to the use of "flipped classroom" (or inverse-blended teaching)for a few of the topics of the course. This option will be completely optional.

Textbook and teaching resource

Different material will be provided during the course. The course has a big practical component for which we will use MATLAB

We will use several books(several chapters in each of them to cover the different topics)

Bibliography:

-Lars Elden, Matrix Methods in Data Mining and Pattern Recognition, SIAM (2019)
-N. J. Higham, : Accuracy and Stability of Algorithms, (2002)
-Horn and Johnson: Matrix Analysis (2012)
-D. S. Watkins, Fundamentals of Matrix Computation, Wiley, (2002)

-D. S. Watkins, The Matrix Eigenvalue Problem: GR and Krylov Subspace Methods, SIAM (2007)
-J. Demmel: Applied Numerical Linear Algebra (1997)
-Per-Gunnar Martinsson &J. Tropp, Randomized Numerical Linear Algebra: Foundations & Algorithms (2021)
-G. Strang, Linear Algebra and Learning from Data, SIAM, (2019)
-I. Ipsen, Numerical Matrix Analysis, SIAM, 2009

-Golub-Van Loan : Matrix Computations (2012)
-Trefethen-Bau: Numerical Linear Algebra (1997) (also 2022)

Semester

First semester

Assessment method

The evaluation of the course has two parts:

1- the development of a small project
2- a small (oral or written) exam. Specifics on the oral or written exam will be given later on during the course.

The students who adhere to the flipped classroom will have the extra vote from their exposition.

The small project could be chosen from a list of projects that will be made available to the students towards the end of the course. The projects will either require students to delve deeper into one of the topics of the course or try to apply many of the techniques for some particular application that can be written as a matrix problem. Students are encouraged to work on the project **in groups** of at most two or three people. The project should be handed four days before the before the date of the small exam. Part of the small exam will be devoted to the discussion of the project, allowing to validate the knowledge and capabilities of the students related to the course.

Office hours

By appointment (that should be fixed by writing an email to me)

Sustainable Development Goals

INDUSTRY, INNOVATION AND INFRASTRUCTURE