UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Basic Calculus - 2

2324-1-E1803M047-T2

Learning objectives

The course aims at giving to the student the basical mathematics tools in order to treat simple mathematical models in economics: after the course the student must have capability in infinitesimal calculus in one variables, with outlines to the calculus in two variables.

Contents

Study of real functions of real variable: domain, sign, intersections with axes, limits of functions, extreme points, monotonicity of a function, concavity, inflection points. Introduction to sequences and functions with two variables.

Detailed program

UNIT 1 - Real functions of a real variable:
Sets N, Z, Q, R. Upper / lower bounded set; intervals; infimum / supremum / maximum / minimum extremum of a set.
Definition of functions and sequences; field of existence; definition of image, image set, counter-image,
counter-image set, graph; use of the analytical expression of a function a a sequence. Use of the graph of a function; injective, surjective bijective function; lower / upper bounded functions; infimum / supremum of a function; minimum / maximum, point of
minimum / maximum of a function; even / odd function; monotonicity of a function and of a sequence. Operations with functions, composition,
inversion. Simple transformations of graphs. Horizontal / vertical translations, horizontal / vertical reflections; partial reflections
horizontal / vertical; rescaling. Composed transformations of graphs.

UNIT 2 - Limits:
Extended real line and neighborhood; definition of internal, external, frontier, isolated, accumulation point; definition of
limit for functions and sequences, right / left limit, limit from above/below; reading limits from the graph. Uniqueness of the limit theorem
(with proof), theorem of permanence of the sign (with proof), squeeze theorem (with proof). Limit computation.
Continuity. Algebra in R*, determined forms, limits of exponential, logarithmic functions,
arctangent. Indeterminate forms, techniques for solving some indeterminate forms (functions
rational / irrational). Asymptotic equivalence and properties. Infinity orders, infinity hierarchies. Negligible function (little-o). Fundamental limits and relative asymptotic equivalences. Indeterminate forms of exponential type and techniques
of solution. Orders of infinitesimal, hierarchy of infinitesimal, little-o. Continuity (from right / left) and
discontinuity. Classification of discontinuities. Indentification of discontinuities from the graph. Horizontal, vertical, oblique asymptotes. Weierstrass theorem with counterexamples, intermediate value theorem with counterexamples, zeros theorem with counterexamples.

UNIT 3 - Derivatives:
Newton difference quotien of a function at a point; derivative function; derivatives of elementary functions;
computation of derivatives. Tangent line equation; continuity-differentiability link, vertical tangent inflection point, cusp, kink. de L'Hopital's rule; Rolle's theorem (with proof) and counterexamples; Lagrange's theorem (with proof) and counterexamples; derivative of the inverse function. Monotonicity test (with proof) and counterexamples; definition of local extrema; stationary point; Fermat's theorem (with proof); definition of critical point; first derivative test for internal extrema. Monotonicity study for sequences. Subsequent derivatives test; first derivative test for boundary extrema; concave / convex function definition; first order test for concavity; second order order for concavity; inflection point. Taylor and McLaurin polynomials; use of Taylor polynomial for limit computation.

UNIT 4- Complete function study and two variables-functions:
General scheme for the study of a function. Analytical and graphical domains for real functions of two real variables;
level curves; partial derivatives, gradient, stationary points

Prerequisites

Set theory. Powers, logarithms, exponentials and their properties.
First and second degree inequalities, rational inequalities, logarithmic and exponential inequalities. Cartesian equations of the line, of the circumference, of the parabola, equation of the line passing through two points. Elements of trigonometry.

Teaching methods

Lessons will be face to face.

Assessment methods

Two-hours written exam with 5 exercises and 3 open questions. The pattern of the exercises is as follows:
Exercise 1: Transformations of graphs of elementary functions;

Exercise 2: Limits;
Exercise 3: General;
Exercise 4: Two-variable functions;
Exercise 5: Complete study of function.

The exam evaluates the formal correctness of the solution steps, the adequacy of the mathematical language adopted, the skills and knowledge acquired during the course.

Once the written exam is passed, the teacher or the student can request an additional oral exam. The oral exam covers the whole course program and can contribute both positively and negatively to the final grade.

The course does not include intermediate tests.

Textbooks and Reading Materials

Textbooks

Guerraggio, A. Matematica 4/Ed. • con MyLab. Pearson.

Additional textbooks

Torriero, A., Scovenna M., Scaglianti, L.: Manuale di matematica. Metodi e applicazioni. CEDAM
Scovenna, M., Grassi, R.: Matematica – Esercizi e temi d'esame. CEDAM.
Monti, G., Pini, R.: Lezioni di matematica generale: funzioni reali di variabile reale, L.E.D.

Further learning material
Lecture notes (available on the e-learning platform).
Texts and solutions of past written exams (available on the e-learning platform).
List of demonstrations that can be requested and examples of theory questions (available on the e-learning platform).

Semester

First semester, first year

Teaching language

Italian

Sustainable Development Goals