SYLLABUS DEL CORSO

Matematica per l'Economia M

2425-1-F8204B021

Learning objectives

The course aims at providing students with some of the basis theoretical tools for analysing those economical models, which are formalized by (continuous time) dynamical systems, as well as with some basic elements in measure theory, which are employed in economical models under uncertainty.

The above skills will enable students:

1) to "read" models evolving with time, whose dynamics are described by ordinary differential equations (ODE), namely to analyze existence and uniqueness of a solution, whenever initial conditions are given, explicitly determine solutions in the linear case and in some other particular cases, search for equilibrium solution, if any, and classify its stability behaviour;

2) to solve optimal control problems by applying the Pontryagin maximum principle and sufficient optimality conditions (e.g. Mangasarian ed Arrow);

3) to solve some of "simplest problems in the calculus of variations";

4) to understand connections in defining an integral with the underlying measure theory and, in particular, to understand the Lebesgue integral along with its basic properties.

This kind of skills should contribute to provinding some of the tools currently employed in the quantitative analysis of phenomena in Economics and Finance, with special emphasis to their dynamical evolution.

Contents

The contents consist of three parts. The first and the second one are strictly intertwined, whereas the third, besides connections with the second part,

provides useful notions for such courses as Finanacial Mathematics M.

In the First Part, basic elements of the theory of ordinary differential equation systems are provided.

In the Second Part, an approach to (continuous time) optimal control problems is presented, along with a solution existence result.

In the Third Part, basic elements of measure theory and of integration theory are provided. As a special case, the Lebesgue integral is considered, with special emphasis to convergence theorems (monotone and dominated).

Detailed program

Part I (ODE):

- Differential equations in mathematical economics, Cauchy problems and related solution notion.
- Reduction to first order ODE of higher order ODE.
- Solving explicitly classes of differential equations: separable equations, linear equations, Bernoulli's equations, homogeneous equations, exact equations.
- Application to specific models (market price dynamics, Solow model of economic growth).
- Global and local solution existence and uniqueness for a Cauchy problem.
- Equilibria and their stability (in the Lyapunov sense, local and global asymptotic).
- Elements for a qualitative analysis of autonomous ODE.
- Linear ODE systems: solution methods and stability.

Part II (Optimal control):

- Problem statement.
- The Pontryagin maximum principle (the linear dynamics case and beyond).
- Sufficient optimality conditions (Mangasarian condition and Arrow condition).
- Applications to economical models (optimal selling strategies, selling maximization).
- The simplest problem of the calculus of variations as a special optimal control problem and its application (an optimal consumption/investment model).
- Existence of an optimal control (Filippov's theorem).

Part III (Selected topics in measure theory):

- Algebra and ?-algebra, generated ?-algebra.
- Measures and their properties.
- The Lebesgue measure on $\mathbf{R}n$.
- Measurable functions and their properties.
- Integral over a measure space and its properties.
- Integral functions and absolutely continuous functions.
- Convergence theorems (Lebesgue's dominated convergence theorem and B. Levi's monotone convergence theorem).
- Riemann vs Lebesgue integral.

Prerequisites

No official prerequisite. Nevertheless, a refreshement concerning the following topics in Mathematics, typically

learnt in basic courses of calculus for undergraduate students, is strongly advised:

- Basic notions about complex numbers;
- Integration of functions of one real variable;
- Multivariable calculus;
- Matrix calculus with basic elements of linear algebra;
- Eigenvalues and matrix diagonalization methods;
- Quadratic forms;
- Convexity for sets and functions.

Teaching methods

The whole teaching activity is done by class lectures.
All lectures are given in person.

During the teaching period, some exercises will be proposed to be autonomously solved by students, in preparation of the exam. Some of them will be then discussed in special sessions by the teacher.

Assessment methods

Students are supposed first to pass a written examinaton (with a mark >=18/30), while a further examination in oral form is optional (up to the student or the teacher). There are no interim assessments.

A written examination esentially consists of the following kind of questions:

- 3 exercise/problem solving;
- discussion in details of one among the models presented as a part of the course contents;
- discussion in detail of some theoretical topics and their application to some specific cases of study.

When answering the above questions, the ability will be assessed in analysing and classifying problems, in choosing and applying provided solution techniques, exactness and completeness in discussing in deepth models as well as the theoretical apparatus presented in the course.

Material for exam simulations is also provided by the teacher.

Textbooks and Reading Materials

Lecture notes and exercises are provided during the course.

Some further reading:

1. **A. Guerraggio - S. Salsa**, *Metodi matematici per l'economia e le scienze sociali*, **G. Giappichelli Editore, Torino, 1997.**
2. **K. Sydsæter - P. Hammond - A. Seierstad - A. Strøm**, *Further Mathematics for Economic Analysis*, **Prentice Hall, Harlow, 2008.**

Semester

The course is scheduled in the second half of the second semester.

Teaching language

Italian.

Sustainable Development Goals

QUALITY EDUCATION | RESPONSIBLE CONSUMPTION AND PRODUCTION